THE MATHEMATICS TEACHER

THE OFFICIAL JOURNAL OF
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS,
INCORPORATED, 1928

VOLUME XXII APRIL, 1929 NUMBER 4

Published by the
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
LANCASTER, PA. NEW YORK

Entered as second-class matter, March 26, 1927, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized November 17, 1921.

[illegible]HE MATHEMATICS TEACHER

[illegible] to the interests of mathematics in Elementary and Secondary Schools

[illegible] This organization has for its object the advancement of mathematics teaching in [illegible] and senior high schools. All persons interested in mathematics and mathe[illegible] teaching are eligible to membership. All members receive the official journal [illegible] National Council—MATHEMATICS TEACHER—which appears monthly, except [illegible], July, August and September.

[illegible]respondence relating to editorial matters, subscriptions, advertisements, and other business matters should be addressed to

THE MATHEMATICS TEACHER
525 West 120th St., New York City

[illegible]SCRIPTION PRICE [illegible] PER YEAR (eight numbers)
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EDITOR'S NOTE

THE MATHEMATICS TEACHER is the only magazine in the United States devoted entirely to the interests of elementary and secondary teachers of mathematics. Moreover, the subscription price of $2.00, which includes membership in the National Council of Teachers of Mathematics, is the smallest fee known for belonging to any national organization in this country.

We cannot believe that the failure of many classroom teachers of mathematics to become members of the National Council and to read the magazine regularly is due to any rigid economy program that they may have adopted. Rather we are afraid that it is due to indifference or because they do not realize how much it means to the cause of mathematics for them to become members of the Council and thus to add their individual support to the great work that we are trying to carry on.

It is perhaps misleading to some teachers to see the word "Council" because it tends to connote smallness in numbers. However, this is not the case. Our membership has now reached 5,000 and we shall continue to grow, but we need to grow faster. In order to continue our progress we need to overcome certain handicaps:

In the first place, members of the Council should send in their annual dues of $2.00 as soon as they are notified to THE MATHEMATICS TEACHER, 525 West 120th Street, New York City. All checks should be made payable to THE MATHEMATICS TEACHER instead of to any individual. We have carried over 500 members on our list for over three months after the first notice of expiration of subscription was sent and then, when no attention was paid to a final notice, we simply had to drop these members from the mailing list. This may mean that they will miss valuable issues and will not be able later to secure them. In order to help members we are printing on the outside wrapper

in which the TEACHER is mailed this month the date of expiration for those whose subscriptions expire in May. Those who do not then send in their dues will be given another notice and if they do not then respond, we shall be obliged to drop them from the mailing list as much as we dislike to do so.

Secondly, we need the enthusiastic support of representatives in certain local groups like Miss Wanenmacher of Buffalo, Mr. Corrigan of Detroit, Miss Constable of Philadelphia, Mr. Austin of Oak Park, Mr. Garland of Boston and others who take a personal interest in the work which the Council is trying to do. Such assistance is helping us to get teachers interested who have never known that THE MATHEMATICS TEACHER existed.

In the third place, we need the help of members all over the country who will write to us for material in advance of their local meetings to use in advancing the work of the Council. At the Philadelphia Mathematics Club meeting on March 9th, Mr. Thilo of the Overbrook High School sold 25 copies of the new Yearbook.

Fourthly, we are trying to get all of our members to send in their names, teaching addresses and positions as they would like to have them appear in the new register of members which we are getting out. Members will find on the last page of this issue a blank for that purpose. Be sure to fill it out and return it at once if you want to be correctly listed in "Who's Who in Mathematics."

Finally, we want to take this opportunity to express our very great appreciation of the support which we are receiving from many members of the Council. THE MATHEMATICS TEACHER is your magazine. We want you to be proud of it and we are doing all we can to improve its outside appearance and its contents. This issue is the Convention number and contains most of the papers given at Cleveland. We hope they will be enjoyed by you as much as they were by those of us who were at the Cleveland Meeting. With the help of Dr. Vera Sanford who was recently selected to be associate editor of THE MATHEMATICS TEACHER the magazine will surely be improved.

W. D. REEVE

INFORMATIONAL MATHEMATICS VERSUS COMPUTATIONAL MATHEMATICS*

By CHARLES H. JUDD

University of Chicago, Chicago, Illinois

I am indebted to Professor G. M. Wilson for the title of this paper. In a book entitled *What Arithmetic Shall We Teach*, which he published through Houghton Mifflin Company in 1926, Professor Wilson says that his findings justify him in recommending a drastic reduction in the amount of computational arithmetic to be taught in elementary schools. He had found in various communities that practical life demands very little training in addition, subtraction, multiplication, and division. It is for the purpose of promoting this idea of drastic reduction that he wrote his book. The reader is almost persuaded that the book should have been entitled, *What Arithmetic Shall We Not Teach.*

While delivering himself of the broadside against arithmetic in the schools, Professor Wilson pauses to admit that there is one phase of the subject with which he has not dealt. This phase he designates "informational arithmetic." He will make some study in the future of informational arithmetic and presumably he will write another book. I suggest that he entitle it, *What Arithmetic I Did Not Know about When I Wrote about Reducing Arithmetic.* It is in a spirit of helpful collaboration on this latter volume that I am speaking this evening.

When one reports Chicago as having a population of three million people, one uses number but one does not add, subtract, multiply, divide, square, or cube the figure; one merely transmits an item of information. When your friend tells you his telephone number or identifies the license tag on his automobile, he uses number and informs you by the use of number; he operates, in other words, in the sphere of informational mathematics rather than in the sphere of computational mathematics.

What is the function of number when it is thus employed

* Reprints of this article may be obtained for 5 cents each from THE MATHEMATICS TEACHER, 525 West 120th Street, New York City.

as a vehicle of information? Evidently number helps the mind to arrange in usable form experiences which would be chaotic and unmanageable if number were not employed. Take, for example, the use of number in the form of dates in history. One knows that Charlemagne lived in 800 A.D. That date stands out in the early history of Europe as a landmark. I am not sure that part of Charlemagne's greatness is not due to the ease of recalling the date of his reign. It was not until 1066 that William the Conquerer won the battle of Hastings and it was later, to be exact, it was in the year 1492 that Columbus discovered America. The pupil who arranges these historical facts along the scale supplied by the number series is not adding or otherwise calculating; he is arranging facts in an orderly sequence. Number is a general system. It was invented by the race as a useful system by means of which anyone who is in command of the number series can arrange all kinds of miscellaneous experiences and thus greatly facilitate intelligent thought about particular items.

Let us consider a few more examples of informational arithmetic. The minutes and hours of the day follow one another in a series that would be very bewildering if each successive unit could not be put into its proper place by means of the number system. Indeed, one may say that the very consciousness of the day as made up of units of time resulted from the perfection of the number system and of certain mechanical devices which permit the number system to be applied to the passing units of time.

What is true of time is true of space. If I want to go to a neighboring town, I ask the automobile association two questions: one relates to the pavement and the other to the number of miles. There is hardly a more common topic of conversation than the number of miles one has to go when he makes this or that trip. Primitive man began by counting the number of days required to make a march. Shorter distances he marked off into stone throws or other crude units. The main point is that he systematized and arranged his experiences. He brought order out of chaos by inventing standards of measure which in turn made it possible for him to use number as the ground plan for all his thinking.

It does not seem at all probable that the advocates of a drastic

reduction of arithmetic in the schools intend to deprive us of our dates and our watches. They seem rather to think that the understanding of number far enough to use dates and tell time is so easy that anyone of average intelligence can master informational arithmetic without very much training. Computational arithmetic is the difficult science for our drastic reducers; informational arithmetic is important but not likely to contribute much to the arithmetic which we teach in the schools.

Let us consider whether informational arithmetic is easy. Evidently primitive man found as soon as he tried to deal with numbers beyond ten or twelve that he had to resort to the device of combining small groups. One can deal with ten things fairly readily but as soon as one is called on to deal with twenty or thirty, it is necessary to say to oneself that these larger numbers are made up of two or three tens. The fact is that the number system itself is a combination of informational and computational arithmetic. The only possible way by which any of us can comprehend large numbers is to resort to that type of multiplication which is implicit in our decimal system of enumeration. Even so, we are not able really to comprehend large numbers. Who among us has any knowledge of such numbers as one billion or even one million? We have to resort to graphic and pictorial devices to give these numbers any real content. McCutcheon drew a cartoon during the war which he entitled "comprehending a billion." In this cartoon he brought out the fact that there have been approximately one billion minutes from the beginning of the Christian Era to the present. The billion is thus reduced to a long chain of historical happenings and we begin to comprehend its meaning. If we think of comprehending a billion as a phase of informational arithmetic, we realize at once how much more difficult comprehension is than calculation. Long before we had any definite idea of a billion such as the historical series helped us to formulate, we were able to perform all kinds of calculations with the number. We could divide a billion into ten parts; we could subtract a million; we could add another billion. In short, computation is extraordinarily easy as contrasted with real understanding of number. Anyone of average intelligence can compute in arithmetic if he has learned a few formal rules, but to think in numbers requires real intelligence.

There is a story about our War Department which illustrates this point. The rations of our small standing army which had to be fed in the years prior to 1915 had long included blackberry jam. The War Department was in the habit of ordering each year so and so much blackberry jam. When supplies began to be provided for the larger army, some clerk in the War Department who was thoroughly trained in computational arithmetic multiplied the usual order by forty and started it on its way. Very shortly the answer came back from the dealers that there were not enough blackberries in the world to fill the order. They offered respectfully the suggestion that the War Department substitute pineapples. It is perfectly easy to multiply anything by forty. In fact, computational arithmetic is so much easier to master than informational arithmetic that the schools have adopted the device of training pupils in the meaning of numbers by having them combine and recombine numbers in examples which look as though they were examples in computation but are in reality devices for training pupils in informational ideas. If one adds 10 and 15 a number of times, one will be more likely to understand 25 than if one has no practice in building up 25.

If computational problems are properly arranged, they will make the pupil acquainted with the attributes of the various points in the number series. Suppose, for example, that one multiplies ten by ten and arrives at a hundred. One gets by this process a conception of the meaning of 100 which he would not have if he merely counted up to 100 by using units. 10 times 10 and 100 times 1 do indeed lead to the same result, but 10 times 10 is a form of analysis of 100 which makes the larger number easy to grasp because both factors are equally comprehensible, while 100 times 1 is confusing. The advantage gained by thinking of 100 as 10 times 10 depends, of course, on a clear conception of ten and of the meaning of multiplication. Granted these preliminaries, multiplication is valuable not merely in itself but as a device for making 100 understandable. If, in like manner, the pupil learns that 100 is equal to 25 times 4 and that it is somewhat more than 12 times 8 and somewhat less than 12 times 9, there will issue from all these calculations a clearer idea of what 100 really means.

It is essential to this comprehension of 100 through an analysis

of its composition that the various formulas of combination be understood and not merely repeated as so many words. To glibly recite the multiplication table is one kind of a mental performance; to utilize it as a ladder by means of which to climb up to the large numbers is a totally different matter.

The schools have not been fully aware of what they are teaching pupils during the long periods of drill in number combinations. Teachers do not realize that examples are clearing up for pupils by a process of induction the true meaning of the number system as a system.

I can express the doctrine which is set forth in the preceding paragraph in another way. The number system embodies certain laws of sequence and relation. What the pupil does when he moves about within the number system through calculations is to make explicit the laws of the system itself. That 100 is the same as 10 times 10 is a lesson in multiplication to be sure, but it is much more than this; it is a discovery of the nature of 100 as a point in the number system.

After using the number system in a great many concrete cases, the learner realizes, as did the race, that this system is an instrument of arrangement and readjustment which can be applied to numerous practical situations in which nature has given no hint of affinity for the number system.

For example, man found in dealing with the surface of the earth that it was far too vast for him to survey in one experience so he divided it up. The early subdivisions were not numerical at all; they were what we call to-day "natural" divisions and finally they were political divisions. The early sectioning of the earth was by rivers and oceans and mountain ranges. The later divisions, that is the political divisions, were determined by the power to defend and govern. It was long after some parts of the earth had been made comprehensible and manageable by such territorial subdivisions that man developed the idea of a complete systematic subdivision. When man became quite universal in his thinking, he drew imaginary lines from North to South and circles extending from East to West. He labeled these with numbers starting with a zero that was dictated so far as longitude was concerned by nothing in nature but by the accidents of his own civilization. Degrees of longitude and latitude can be used for purposes of calculation but the great

achievement in this case was the application of the number scheme to a reality which nature presented in an utterly unsystematized form.

What one can say about the geographical use of number can be said about all kinds of space measurement. Man has adopted standard units which nature never suggested and has applied them to continuous space. He took the span of his hand and the length of his foot and he marked off space with these units of measurement for the purpose of reducing space to a manageable form. Number is the great universal systematizer.

I am always impressed with the shallowness of the reasoning which has been widely accepted in recent years according to which it has been concluded from rather trivial experiments that the mind of man does not transfer its ideas and forms of thinking. The number system is a universal scheme which man has learned to transfer to almost everything. If man wants to make the location of houses on the streets of his city quite definite, he numbers them. If man wants to designate the members of the police force so that he can readily refer to them, he numbers them. He numbers the convicts in his prisons and the customers waiting for their turns in the barber shop. If there is anything which cannot be numbered from unhatched chickens to one's debts, it would be interesting to know what it is. Transfer of the number system from one situation in which it was learned to other situations which need to be put in order is so common that the psychologists who deny transfer have overlooked it just as most people overlook the obvious laws of nature in the most familiar experiences.

Nor can the psychologists who tell us that transfer of ideas is uncommon escape from their impossible position by saying that transfer takes place only when there are present identical elements. Of course there are identical elements present after the transfer has taken place. Number is present where number is present. The identical element is exactly the subject under discussion. To say that the identical element was there from the beginning is to overlook the fact that the mechanics of the transfer took place in a world which had no number in itself but was invaded by a mind which was informed and equipped with number. Number is a device for arranging experiences; it is one of the most general of all generalizations. It is not present

in immature minds. It must be built up in one's thinking by all kinds of use in simple situations. When it has been developed in a mind by the counting and adding of blocks, it can be perfected by counting and adding coins and other possessions until finally the mind that was immature and without number comes which the radicals who want drastically to reduce arithmetic have overlooked except as they admit the presence in the world of informational arithmetic. I am reminded of the biblical parable about the stone which the builders at first rejected but which they later found to be suitable for the head of the corner.

Erratum: See insertion of 3 lines page 332 of this volume.

At this juncture I shall turn abruptly from the discussion of arithmetic to higher mathematics. I shall attempt to introduce this part of my discussion by what will, I am sure, seem at first to be a digression. The apparent digression calls your attention to the fact that we derive from our experience a great many attitudes of which we are hardly aware. Buried deep in our personal experiences are certain prejudices, such as party-political faiths which color all our thinking. Most of us would doubtless assert that we are wholly unbiased if we were interrogated regarding our political attitudes. We have prejudices but we have long since left behind the experiences which gave rise to them and in this sense we are not aware of the presence of the prejudices. In like fashion, to continue the apparent digression, we carry through the day and attach to all sorts of miscellaneous situations feelings of depression or feelings of exultation which originated in some trivial incident in the early morning. In short, we are constantly carrying over mental attitudes having remote origins, which origins are entirely lost to view.

The interpretation of the foregoing digression is as follows. If you ask a person who studied algebra in his high-school course whether he uses his algebra in ordinary life, the chances are about ninety out of a hundred that you will be told emphatically that algebra never functions at all in ordinary life. This means that computational algebra does not function. Very few people do any factoring after their 16th year of life. Very few people have occasion to write out the binomial formula. Computational algebra is almost totally useless in ordinary life. This does not supply any valid ground for the assertion that algebra is wholly lost. Is there any informational algebra? If there is

informational algebra, it may be that its possessor will continue to attach it to all kinds of miscellaneous experiences even though he is unable to assign the informational algebra to its true origin.

Informational algebra there certainly is. It has to do with such matters as the nature of the equation and with positive and negative numbers. No one can pass through the rigorous drill of higher mathematics without carrying away a notion of the equation which is totally different from that which is present in the mind of a person who has never manipulated equations. Computational algebra is the rudimentary algebra. The pupils in the schools are not concerned in the long run with the mere computations but they are concerned with the absolute and invariable integrity of the equation. Equations always balance and he who has once comprehended this fact as a result of much manipulation of equations is informed. He has information which will help him in physics and chemistry, in economics and in ordinary life. The situation here is exactly the same as in arithmetic. We pass pupils through the computational stage for the purpose of bringing them to the higher levels of information. Informational mathematics will be greatly reduced if the critics of computational mathematics have their way and go on determining the amount of mathematics which shall be taught in the schools by measuring the amount of computation performed by the ordinary man.

My purpose in drawing the contrast between computational mathematics and informational mathematics has not been merely to reveal what I believe to be the fallacy which has led to the current tendency to reduce the amount of mathematics in the schools. My purpose has been rather to accept the opportunity which your invitation has given me of preaching a sermon which has been very much on my mind of late. The oversight which has been committed by the enemies of mathematics who ignore informational mathematics is to be explained in no small measure by the ineffectiveness of much teaching of mathematics which obscures the informational aspects of the science. The textbooks in mathematics are written in a style and in a form which is anything but stimulating to the mind seeking information. These textbooks do not select for emphasis those aspects of mathematics which are informing. The textbooks are computational books. Some pupils, thanks to the generalizing ability

of the human mind, arrive at a knowledge of the number system and of the equation and of the other aspects of informational mathematical thought through the exercises which are on their surface purely computational. All pupils could be greatly helped in performing their generalizations if the mathematicians would stress the higher informational types of experience which their science is designed to cultivate.

The desirable innovation in teaching which I am advocating will not be secured by talking about informational mathematics. The content of courses in arithmetic and algebra must be changed. There are an infinite number of excellent examples in the world of the informational use of number, but these examples need to be discussed. The pupil needs time to understand informational arithmetic. His textbook ought to be enlarged to about twice or three times the present size. The textbook should tell in clear statements which can be read by the pupil what number really means.

You will perhaps ask me why I advocate a radical change in the long established methods of teaching these time-honored subjects. My answer to this question is ready. The statistics show that there are more failures in arithmetic than in any upper grade subject in the elementary school and there are more failures in algebra than in any freshman subject in high school. The sinfulness of this condition is thought of by many teachers of mathematics as a virtue. Mathematics teachers sometimes try to cover up their own inefficiency by saying that pupils are stupid, that only the brightest minds can add, subtract, multiply, divide, and factor. The fact is that these mechanical operations can be performed by machines and have been performed by persons whose general intelligence was below normal. The failures in mathematics are in very large measure the result of incompetent teaching.

I do not appear before you in the spirit of an alarmist, but I must frankly report what I see as the signs of the times. If the mathematicians are incapable of discovering or unwilling to discover and utilize the illuminating informational aspects of their science, our schools will pass through a period of reduction of the amount of mathematics included in the curriculum. The traditional computational introduction to informational mathematics is clumsy and ineffective. The hard crust of

tradition which for generations has covered and obscured the real essence of mathematics must be removed. When one recognizes the historical fact that a long procession of textbooks in mathematics has followed antiquated models so faithfully that it is almost impossible to tell them apart, one realizes that a revolution may be necessary to disturb the complacent slumbers of the teachers and textbook makers in this field. Perhaps American schools will have to be deprived for a time of the full benefit of the kind of mathematics now current in order that mathematicians may be persuaded to mend their ways.

My sermon is ended. My hope is that what I have said may stimulate someone to prepare more informing books on mathematics before the revolution comes and deprives us further of mathematics in the schools.

The relations with which the mathematician deals seem to be a part of the very foundation of the world we live in, so that we have discovered that, if any proposition that holds true of experience is elaborated in accordance with the rules of mathematics, the conclusions thereupon reached will also hold of experience. This fact about our universe, and the additional fact that the quantitative methods of mathematics admit of the utmost accuracy and precision of formulation, explain why in each of the fields we have looked at so far mathematics is so fundamental. It does seem to be true that the more highly developed a science becomes and the more knowledge we gain about the relations between its object, the more its beliefs tend to fall into mathematical methods. So true is it that a science is successful just in so far as it is able to formulate its beliefs mathematically, that many men have naturally come to think that in mathematics is to be found the exemplar of all true knowledge.

Introduction to Reflective Thinking by Columbia Associates in Philosophy (Houghton Mifflin, 1923).

THE PERMANENT NATURE OF THE NECESSITY FOR MATHEMATICS IN THE SECONDARY SCHOOLS

By LOUIS C. KARPINSKI

University of Michigan, Ann Arbor, Michigan

Some years ago I heard a prominent educator, or, at least, professor of education, assert that the day would come when we would in our schools give special courses in the arithmetic for the plumber, the arithmetic for the carpenter, the arithmetic for the housewife, and the like. In some most mysterious way these poor children were to be labeled "plumbers" or "carpenters" or "housewives" and the education given would be fitted to the label. It is the glory of our American schools that the children are not ticketed and labeled, and the ultimate destiny of the great majority lies in the hands of Providence and, mayhap, the teachers.

The arithmetic as we know it and as we teach it applies equally well to the needs of the carpenter and the physician, to the housewife and the nurse, to the plumber and the professor. By the diversified problems of the arithmetic the child is prepared for the particular need, which may be to him business and at the same time given a sympathetic understanding of the problems of other professions. To assume that there is need in our elementary schools for specialized instruction in arithmetic for each vocation is nonsense.

Probably the future will see less necessity for much of this so-called business arithmetic in the lower grades and will include some of the vital material in later years of the high school course. This will bring the information and the methods to the pupil more nearly at the time when the information is likely to be used.

Intelligent appreciation of simple arithmetical facts is desirable for every citizen of the American Commonwealth. In business ventures arithmetic plays so large and vital a rôle that inability to grasp the mathematical situation dooms the individual to failure. Even in the ordinary financial affairs of the home, arithmetic properly applied leads to success, and improperly applied leads to failure. For the great majority of individuals arithmetic is an indispensable tool.

As so large a proportion of our children continue in school beyond the eighth grade, it becomes increasingly desirable that the mathematical courses of the high school should make definite and systematic instruction in arithmetic, notably in connection with the algebra and geometry. In the geometry and algebra the arithmetical illustration often gives the practical application which connects with the world of affairs and which illuminates the theory.

For a number of years now, each year has seen an increasing proportion of the high school students entering college. No other country in the world sends so large a proportion of young people to college. Accurate statistics as to the reasons which motivate these young people are not available. However such investigations as have been made show that a large percentage of our freshmen have not fixed upon a choice of vocation upon entering college. For an increasing number of professions mathematics is a prerequisite. In consequence the high school student should be given the necessary instruction so that the door to the professions will not be closed. Again and again the student who has desired to enter medicine or to study engineering, has been compelled to select some other vocation as the extra year to make up the entrance requirements has proved an insurmountable barrier. Teachers of mathematics would do well to read and ponder carefully the chapter in the National Committee Report (on The Reorganization of Mathematics in Secondary Education) on the "Change of Mind between High School and College as to Life Work." The requirements of college entrance and the prescribed preparation for many professional courses will for years to come in America demand definite instruction in Mathematics for the great majority of high school students.

Entirely apart from this questionable motive of college entrance, the mathematical view point, even that of algebra and geometry, is increasingly desirable for the cultured individual. No one disputes the desirability of instruction in history. and literature, entirely apart from any bread-and-butter motives. Yet the mathematics represents the individual as part of the thinking world not only of every region but of every age since history began. In this study the student must be made to feel himself heir of all the ages. The consideration of the quadratic

equation, the consideration of the circle and parabola and ellipse, the sine and cosine and tangent functions, these are fundamental whether the mathematics is studied in China or New York. For more than a thousand years these ideas have been fundamental in practically every system of education worthy the name. And we who teach mathematics are convinced that if communication is ever established with other planets, the developments certain to be found there are geometry and algebra, trigonometry and the calculus. If there is intelligence on Mars there is mathematics.

Recently in a controversial article an American astronomer asserted that we are too much bound in science by the formula. Another scientist was quoted in the public press as stating that from the mathematics we can get nothing which we do not put into the mathematics. These gentlemen are exceptions in the world of science. Evidently their knowledge of the progress of science is somewhat limited. Take away from the phenomena connected with the freely falling body the formulas and you have left a mass of statistics beyond the power of any human being to organize. The formula and almost the formula alone organizes this mass of statistics, not only here but in hundreds of situations. The formula does in general two things. In the first place the formula organizes and accounts for the observed facts. By the formula the human mind is enabled to control tens of thousands of situations which otherwise are not subjected to a law within the power of the scientist. But beyond this the formula extends the observations and suggests new types of observation which give greater control over nature.

In the case of the freely falling body, the formulas $s = 16t^2$ and $v = 32t$ give a mass of information which would otherwise be incoherent to all but a gifted few. The differential and the integral calculus show the connection between the two formulas. However, beyond this the mathematics suggests to the physicist that analogous equations hold in the case of a body shot vertically into the air—so we get equations like $s = 800t - 16t^2$ and $v = 800 - 32t$ for the motion of a bullet shot vertically upwards with a velocity of 800 feet per second. Then it is seen that these formulas can be extended to the bullet shot at an angle with the horizontal. Some one will

venture, doubtless, that these formulas are only approximate, neglecting the resistance of the air. But the scientist knows that you must begin with this simpler situation. If this cannot be handled mathematically there is no hope for the more complicated situation.

In the case of the phenomena of sound Helmholtz found that new formulas suggested new types of acoustic instruments producing results not previously known to physicists. That is the real proof of the mathematical formulation extending the results of observation.

Kepler contemplated the conic sections with whose properties as a student of Greek science he was familiar. Kepler saw that a modification of the Copernican theory explained, so to speak, a whole series of observations which were unexplained by the Copernican theory. In consequence Kepler was able to state that planets move in ellipses with the sun at one focus and the further laws of planetary motion. Kepler was able to do this because the Greeks out of pure love of geometry had advanced the study of the parabola, the ellipse and the hyperbola.

Newton appears upon the scene after the mathematical methods of the analytical geometry had been perfected and at a time when the ideas of the calculus were germinating not only in Newton's mind but in the minds of numerous scientists in England and on the continent. The studies by Cajori show what a host of men were and had been occupying themselves with these ideas. By the aid of the new machinery Newton was able to grasp the stone falling to the earth, the moon revolving about the earth, and the earth revolving about the sun as phenomena subjected to the same law. And since that day the poor astronomers have been "bound," so to speak, by that law; or have they been freed by that law? Even to-day the new law of an Einstein starts from the old law of Newton. Nor does any real scientist suspect that for generations to come the formulation given by Newton will not control more than 99 percent of the astronomers' work. The new law itself is justified again not primarily by explaining a few phenomena not explained by the Newtonian formula—rather the acceptance of the new law hinges upon the prediction of certain phenomena not previously observed by the "watchers of the skies."

Our old friends, the linear and quadratic functions in one and two variables, control a host of situations in the physical world. In the correct auditorium, $y^2 = 4ax$ is the equation which determines the form of the hall so that the sounds emanating from the stage should act properly. The reflector of your automobile headlight and the sound-detector of distant submarines operate under the same law which governs the falling body. The man who lays a street car track, or, not to take an extinct institution, let us say railroad track, needs to know that the amount of expansion is a linear function of the temperature. Otherwise there will be difficulties.

While Newton profited by the pure love of science of the Greeks, he himself studied for pure love of science the binomial expansion $(1+i)^n$. To-day the whole structure of our financial world is based upon this formula and its direct extensions. Life and fire insurance premiums involve the theorem. Even the sufficiency of a nickel or seven cents for a bus fare, and the price of a sandwich or a pair of shoes, hangs upon the formula. For in every business the machinery and the buildings that are used must be paid for by the products of the business. In the estimate of the cost of production the cost of renewal of the machinery involves the binomial formula. So that when a child is taught that a dollar placed at 5 percent compound interest for ten years amounts to $(1+0.05)^{10}$, the child begins an acquaintance with a formula whose power extends over a hundred domains.

Somewhere in our instruction in elementary mathematics we must include the applications of elementary mathematics which control so much of the modern world of finance, of commerce, and engineering. The pure love of science may have been sufficient for the ancient Greeks but we are presenting this material not only to a far larger public but also to youth somewhat jaded by too much automobile, radio, phonograph, and movie. Incidentally no one of these developments can really be understood without mathematics. The comprehension of the universe and the civilization in which we live is impossible without mathematics. The desire to know and to understand is that which distinguishes man from the animals. Even the dog likes a good roof and good food, but man desires to understand what makes the machinery go. We have the mathematics involved then in the world about us. Not in any indefinite way

like the air we breathe and the sunlight, but definitely in the attempt of rational beings to understand and control the forces of nature and of society. The circumstances of life which require counting and measuring are almost infinite in number. Some few individuals may escape the actual necessity of anything beyond the simplest mathematics. However, no one without mathematics can have any real comprehension of the laws of physical forces nor of the laws back of commerce and engineering.

To you teachers of mathematics it is a duty to familiarize yourselves with these applications of mathematics to physics and chemistry and biology and commerce so that you may give to the enquiring student a reason for the faith which is in you, the faith and conviction which we have that mathematics is an inevitable development in a universe where there is intelligence.

Something has been said about the use of mathematics in physical science, the mathematics being regarded as a weapon forged by others, and the study of the weapon being completely set aside. I can only say that there is danger of obtaining untrustworthy results in physical science, if only the results of mathematics are used; for the person so using the weapon can remain unacquainted with the conditions under which it can be rightly applied. . . . The results are often correct, sometimes are incorrect; the consequence of the latter class of cases is to throw doubt upon all the applications by such a worker until a result has been otherwise tested. Moreover, such a practice in the use of mathematics leads a worker to a mere repetition in the use of familiar weapons; he is unable to adapt them with any confidence when some new set of conditions arise with a demand for a new method; for want of adequate instruction in the forging of the weapon, he may find himself, sooner or later in the progress of his subject, without any weapon worth having. —Perry's Teaching of Mathematics (London, 1902), p. 36.

MATHEMATICS AND THE FUTURE

By CHARLES N. MOORE

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In recent educational literature and discussion much has been said of the modernization of education. I take it that this phrase usually means the adjustment of education to what the speakers or writers conceive as the needs of the modern world. But what is the modern world? Most thinking persons will concede that the world of to-day is different in many important respects from the world of five years ago, or even of one year ago. While we are engaged in the process of constructing an educational system that will exactly fit a world as we have visioned it, the world as it is will have changed into something quite different.

What security have we then in framing a scheme of education for its prime purpose, training for life, that it will not resemble a house built on shifting sands? As I see it, we have two important safeguards. In any set of objects that are in a state of change, we can usually find certain elements that remain fixed. The mathematician designates such elements as invariants. Moreover, in any change with time that is of the nature of what the mathematician calls continuous variation, we can usually determine approximately the variation in the near future by means of the observed variation in the recent past. This process, known technically as extrapolation, has certain hazards, but it is the safest guide we possess in attempting to predict the future.

If we wish then to select as essential elements of secondary school education the subjects that give best promise of being useful to our future citizens, we must consider the entire past history of the civilized world and its probable future history during the next forty or fifty years. If certain fields of study have at all periods of the world's history shown their importance in contributing to the progress of knowledge and the advance of civilization, then they surely rank as invariants among human needs, and their importance in education is incontestable. If the changes that have taken place in recent years suggest a

steadily increasing need for a certain type of training, then we owe it to the rising generation to give greater stress to that type of training in all our educational enterprises.

It follows from these considerations that any subject of instruction which demonstrates its importance by both of the tests we have suggested, occupies a unique position in the educational field. There can be no question as to the desirability of giving to all our students the maximum training in this field that they are able to assimilate. Those who hold the contrary thesis thereby show themselves incapable of reading the lessons of the past and blind to the changes that are taking place in the world about them.

It will be the purpose of my talk to-day to indicate in a broad way the well-founded claims of mathematics to the unique position I have just described. To treat the matter in detail would require volumes, so I shall have to content myself with pointing out a few salient facts that will serve to illustrate the general line of thought. I will first deal with the past as a whole and endeavor to show how mathematical knowledge has been of first importance in determining the nature of the universe in which we live and in carrying out many of the scientific and technological investigations that were essential in attaining the material prosperity and the comfort and luxury that are characteristic of the present age. I will then consider the more recent past and point out some of the newer applications of mathematics that serve to astonish those who have not reflected deeply on the great historical trend in all fields of knowledge toward quantitative results and quantitative relationships.

Let us then first consider the achievements of mathematics during the entire historical period. We find that mathematical methods and the results obtained by them are so interwoven in our modern civilization that if it were possible to withdraw the contributions due directly or indirectly to mathematics very little would remain. Our present system of writing numbers, which enormously facilitates numerical calculation, was at the time of its inception a mathematical discovery of the first importance. If any one doubts its essential usefulness in modern civilization, let him try to multiply together two numbers greater than one hundred, using Roman numerals throughout the process. In the present age those enterprises where a great deal of

numerical calculation is necessary obtain far greater facility by the use of tables and calculating machines, both of which were the inventions of mathematicians. It is worthy of note that Pascal and Leibnitz devised the first modern calculating machines more than two hundred and fifty years ago, and if the general level of mathematical culture had been sufficiently high it would not have required more than two centuries to reach the point where proper use of this great discovery was being made.

Our knowledge of the most fundamental facts with regard to the universe in which we live is very largely due to mathematical investigation. Let us consider very briefly how this knowledge was obtained. It was natural for an inhabitant of the earth to consider himself at the center of the universe and to suppose that all the heavenly bodies moved about this center. This was the point of view of the Greek school of astronomy, but a detailed study of the motions of the planets on this basis led to hypotheses of greater and greater complexity so that the so-called Ptolemaic system was discredited by mathematical investigation. The most natural substitute was the supposition that the earth and all other planets rotated about the sun, but it remained to determine the nature of their orbits. The simplest hypothesis was that they were circular, but the conclusions from this hypothesis did not agree with the observed motions. The closed curve next in simplicity to the circle is the ellipse, and its properties had been thoroughly studied by the Greek mathematicians without any idea of making a practical application of them. Equipped with a knowledge of these properties, the mathematical astronomer Kepler was able to determine that the planets move in ellipses with the sun at one focus and to formulate in addition two further laws with regard to the their rate of motion and their distance from the sun. This was at the beginning of the seventeenth century. During the latter part of this century Newton was able to verify his hypothesis as to the law of gravitation by showing that Kepler's three laws were consequences of this more fundamental law. He was able to do this by means of a new and powerful mathematical method which he had just invented, namely the calculus.

Newton's great discovery, which would have been impossible without the use of the highest mathematical knowledge available

at his time, represented the longest step that man had taken in the comprehension of the inner nature of our universe. For more than two centuries his successors in the field of celestial mechanics were engaged in studying the various ramifications in the theory that logically follow from this one fundamental law. It was not until the second decade of the twentieth century that it was found possible to arrive at a more general view of gravitational phenomena. And it was only by making use of some of the noteworthy advances in pure mathematics made during the nineteenth century that Einstein was able to elaborate his remarkable theory.

About the middle of the last half of the nineteenth century the British physicist, Clerk Maxwell, concluded from certain mathematical investigations that electrical disturbances were propagated in waves. Somewhat later a German physicist, Hertz, verified experimentally the existence of these waves. If these fundamental scientific discoveries had not been made previously, the invention of wireless telegraphy and the radio could not have been achieved in our day and age.

I have given merely a few of the striking instances in which mathematical methods have in the past contributed vitally to the progress of civilization. Some of the illustrations involved mathematical methods that are beyond the scope of high school courses, but they were chosen advisedly to illustrate the fact that the higher developments of mathematics are as fruitful in applications as the most elementary facts of arithmetic. Hence the more mathematics a given individual knows, the greater will his opportunities be of making useful applications of his knowledge.

If we turn now from the study of the past to the consideration of the present and the future, we shall soon see that the opportunities for applying mathematical knowledge are more numerous and varied than at any previous time and that their number and variety are rapidly increasing. In our previous illustrations of the applications of mathematics to science we have drawn only from the fields of astronomy and physics, and in the past such applications were largely found in these fields. But it is the tendency of each science to become more precise as it develops, and whenever it reaches a certain stage of precision mathematical methods can be applied to it to great advantage. When Comte wrote his Positive Philosophy, he ex-

pressed doubt as to the possibility of applying mathematics to chemistry, and he openly asserted that it was absurd to suppose that you could ever apply it in such complex fields as meterology, biology, and sociology. Now-a-days you cannot even read the article on chemistry in the Encyclopedia Britannica without encountering the methods of the calculus, and important applications of mathematics have been made in each of the other fields mentioned above and in still other fields such as economics, psychology, and education. As the knowledge in each of these fields is continually gaining in precision, it is obvious that the possibilities for applying mathematical methods must rapidly increase in number in the near future.

In order to give you some idea of the fundamental importance of some of the recent applications of mathematics in the various fields above mentioned, I will briefly describe a few of them that are especially noteworthy. The first one is selected from a field where most people would least expect to find an application of higher mathematics, namely the field of medicine. Before the outbreak of the late war Dr. Alexis Carrel, of the Rockefeller Institute of Medical Research, had noticed in the course of some of his experiments on animals that the rate of healing of a wound seemed to be approximately proportional to its surface area. During the war, as you probably know, he had charge of the large base hospital at Compiègne and therefore had ample opportunity to verify the correctness of his original observation. He soon came to the conclusion that the relationship between the rate of healing of a wound and its surface area could be represented by a mathematical equation. Not having time to work out the details himself, he turned the problem over to one of his colleagues, Dr. P. Lecomte du Nouy, who found after some investigation that the area of the wound surface at any given time could be expressed in terms of the area at the time of the first observation, the interval of time elapsed since that observation, and a certain quantity i known as the index of the wound. This quantity i can in any given case be determined from the first observation and a second observation taken four days later. Once i is determined it is possible to plot the curve of healing and predict the future progress of the wound. It was found by Dr. du Nouy that the curve thus obtained agreed in most instances with the actual course of healing. When there was

any marked departure from the theoretical curve it was due to some abnormality in the particular case such as infection in the wound. This made it possible in many cases to detect the presence of infection by comparison with the curve before it was possible to determine this fact by direct medical examination. Moreover, it was found by further investigation that the quantity i mentioned above depends on the age of the patient and the area of the wound surface. Hence for a given age and a wound of given size there is a characteristic value of i. A departure from this value reveals the fact that the general condition of the patient is not normal. Further important applications of the curve of healing have been made by other workers, but I have not time to describe them here. Those who are specially interested in the matter can find the material in recent volumes of the Journal of Experimental Medicine.

We turn next to two other recent applications of mathematics taken from the field of economics. One of the most obvious of economic phenomena is the alternation of periods of prosperity and periods of depression, or, to phrase it more technically, the existence of economic cycles. One of our prominent economists. Professor H. L. Moore, of Columbia University, set himself the task of finding the cause for these cycles and determining as accurately as possible the law which governs them.[1] His starting point was the well known doctrine of the dependence of all forms of economic life on agriculture. Since the yield of crops depends largely on the state of the weather, the next logical step was to examine meteorological records for the existence of cycles in weather conditions. In order to bring the labor involved within reasonable bounds, Professor Moore considered only rainfall data and limited himself to the Mississippi valley. By the use of a method known as harmonic analysis, originally devised over a hundred years ago in order to study the flow of heat he was able to detect the presence of two principal cycles in the rainfall of the Mississippi valley, one of length 33 years and the other of length 8 years. Then, taking four standard crops, potatoes, hay, corn, and oats, he was able to show that the yield data exhibited the same cycles as the rainfall data. Taking the production of pig iron as a measure of the activity of industry, he found that this production exhibited the same cycles as the

[1] *Economic Cycles: their Law and Cause* (New York, 1914).

crop yields with a lag of about two years. Finally, the curve representing the movement of the general price level showed the presence of the same cycles with a lag of four years. Thus we have established the cause of economic cycles and the law which governs them. It is obvious that many applications can be made of this knowledge, particularly in connection with the problem of unemployment.

Somewhat more recently the same economist set himself the task of forecasting the yield and the price of cotton by means of a study of weather conditions in the cotton belt.[2] In this instance his principal mathematical tools were the methods of mathematical statistics, largely due to Professor Karl Pearson and his coworkers, who devised them primarily for use in the fields of biology and anthropology. By a detailed statistical analysis too complex to be outlined here, Professor Moore determined from the data of official reports what weather conditions at particular times were most favorable for the growth of cotton. To summarize his results briefly, the main requirements are that May shall be dry, June warm and dry, August cool and wet, the latter requirement being the most important. By comparing his predictions, made by this method on the basis of weather reports only, with those made officially by the Department of Agriculture and based on detailed reports of actual crop conditions from every section of the cotton belt, he has shown that his method yields results that are distinctly more accurate. Furthermore, by combining his predictions as to yield with the law of demand for cotton, also obtained by mathematical methods, he was able to forecast the actual price of cotton more accurately than the Department of Agriculture, with all its vast machinery for obtaining detailed crop reports, was able to forecast the yield.

The final illustrations of recent applications of mathematics in new domains of investigation are chosen from the fields of meteorology and scientific agriculture.[3] The methods employed are substantially those of the previous illustration, that is to say, the methods of mathematical statistics; the results obtained are of equal if not greater practical importance. The investigations in question are due in the main to Professor J. Warren Smith, of the U. S. Weather Bureau, who set himself the task

[2] *Forecasting the Yield and the Price of Cotton* (New York, 1917).

[3] Cf. "The Mathematician, the Farmer and the Weather," by Thomas Arthur Blair, *The Scientific Monthly*, Vol. XI (1920), p. 353.

of determining as completely as possible the influence of weather conditions on crop yields. By an elaborate statistical analysis he was able to show that there are certain critical periods in the growth of each crop, and he was able to locate these with great exactness. For example, if we are considering the yield of corn in the state of Ohio it is found that the period comprising the first ten days of August is the time when rainfall is most important. If the necessary rainfall takes place at this time you are practically certain of a good crop; otherwise you will have a poor one. In other crops it may be temperature that is more important, and the critical period in any case will vary with the crop. The application of all this to scientific agriculture is fairly obvious. The climatic conditions in different parts of the United States are fairly well known from previous observations. Then, if we determine the critical periods of the various crops and the kind of weather needed, we are able to determine the likelihood of the weather conditions being favorable for that crop in a given locality. In other words we can determine whether it will be profitable to raise that particular crop in that particular district. Furthermore, by using an earlier or later variety, by varying the time of planting and other similar methods, the critical period may be advanced or retarded so as to take better advantage of probable weather conditions. Also moisture may be conserved in the soil by cultivation if we know the period when it is needed, and many additional applications of the knowledge we have gained will occur to the scientific farmer.

Within the domain of meteorology proper the methods of mathematical statistics have enabled metorologists to establish curious and unexpected connections between weather happenings in various parts of the globe. For example, a cold day in London is apt to be followed by a warm day in Cairo, Egypt; a light rainfall in Chile from May to August indicates the probability of unusually heavy floods in the Nile from July to October; and so on. You can readily see that we are on our way to an exact science of meteorology, and it is by the use of mathematical methods that we shall arrive at that goal.

The examples I have presented of recent applications of mathematics in entirely new fields are merely a few typical illustrations of a wide general trend. In every field of modern inves-

tigation the importance of precise results is being more and more clearly realized. A very distinguished British scientist of recent times, Lord Kelvin, made the remark "When you can measure what you are speaking about and express it in numbers, you know something about it; when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind." He has also said "All great scientific discoveries are but the rewards of patient, painstaking sifting of numerical data." The essential soundness of these statements is coming to be widely realized by investigators of all types. Hence the remarkable increase in the use of mathematical methods.

Some of you may object that what I have said of recent tendencies in various fields of investigation is no proof that every high school student should study mathematics, since relatively few of them will become investigators. Even if we grant the truth of the latter statement, we must acknowledge that we cannot always determine which is the future investigator, and it is of first importance to society that all investigators be properly trained. But aside from this, I contend that most of our high school graduates should become investigators to the extent of their ability, or else our system of education is not realizing to the fullest extent one of its primary purposes, namely the advance of civilization. The great leaders and pioneers of research should have the cooperation of hosts of lesser men who contribute their mite to the general task, in order to accomplish the best results. In October, 1918, our newspapers were carrying full page advertisements, stressing the crying need of men trained in mathematics to serve as artillery officers. I think the previous illustrations will have shown you that mathematics is fully as useful in the activities of peace as in the activities of war. But we don't see any advertisements in the papers at present, urging our youth to study mathematics. Professor Vernon Kellogg, in an address on "The University and Research," asks the question "Do we have to have war to be capable?" I must confess that it sometimes looks that way.

I have chosen my illustrations of recent applications from the field of science rather than the field of commerce, industry, and finance. This was done advisedly, because the more striking and far reaching applications are in the former field. The reason

for this is that the great majority of men who undertake scientific work have some training in mathematics beyond arithmetic, and some of them have a very considerable training. If the average of mathematical training were as high among the men in the business world as it is among the scientists, we would undoubtedly have a much larger number of vitally important applications in the commercial fields. As it is we already have a considerable number. For example the determination of the premium on a life insurance policy involves a large amount of mathematical work, as you probably know. Likewise the valuation of bonds and the solution of many other related problems in the investment field require the use of algebraic processes of varying degrees of complexity. The methods of mathematical statistics, which have been found so valuable in both the natural and the social sciences, are being used more and more widely in commercial fields. They have also proven useful there and will undoubtedly be far more useful when a larger proportion of statistical workers have a considerable amount of mathematical training. It is a rather common error to think that statistical work of some value can be accomplished by workers who have no training in mathematics beyond arithmetic. This is a fundamental mistake. The sort of statistical work that such people can do is much better done by calculating machines. The really valuable work in statistics is statistical analysis, and that requires a certain amount of mathematical training. When persons without such training attempt this type of work they practically always commit gross errors.

We have now considered in a brief way some of the past and present contributions of mathematical ideas and mathematical processes to the advance of science and the progress of civilization. We have also shown by a few typical illustrations that the field of application of mathematics is at present widening at an extremely rapid rate. It should be apparent then that the future citizen who engages in some form of intellectual activity is very apt to have need of mathematical knowledge and mathematical training. If he has received no training beyond arithmetic in the course of his education, he encounters almost insuperable difficulties in making up his deficiencies. Mathematics needs to be acquired in youth; it is a field in the study of which increased maturity never compensates for the lack of that mental plasticity that is characteristic of the young.

In view of the importance of mathematical training for the citizen of the future and the necessity for having this training in youth, it is apparent that we have excellent reasons for including mathematics in all high school courses. The only possible justification for not so including it is that we might thereby exclude other material of greater or coordinate importance. I think that it can readily be shown that this is not the case. The history of the past, present conditions and present tendencies all indicate very clearly that there are two great methods for finding out truth, the method of observation and experiment and the mathematical method. Every high school student should therefore have some contact with each of these methods, that is he should study at least one of the natural or physical sciences, and he should have some training in mathematics beyond arithmetic. No other form of training, except that which deals with self-expression in one's native tongue, can present a valid claim to being of coordinate importance in the present age.

Some may reply to the argument I have developed as follows. We grant that it is important for the high school student to have mathematical training, but how about the boys or girls that cannot learn mathematics? Are they on that account to be deprived of the benefits of a high school education in other fields? I am yet to be convinced that there are boys or girls whose mentality justifies their receiving a high school education and who at the same time are not able to learn mathematics. Elementary mathematics is merely ordinary common sense with a condensed notation, which of course requires some effort to learn, but which enormously repays that effort by the gain in technical facility that results. There is no more misleading phrase in current use than the expression frequently heard that "Some people have mathematical minds and some do not." Such a statement reveals a profound misconception, both of the nature of mathematics and of the nature of human intelligence. We find that some people have the physical qualifications which enable them to play an excellent game of basket-ball, whereas others are not so endowed. We do not therefore say that a person of the first group has a basket-ball body, while one of the second group does not. Yet such a statement would be quite as sensible as the one to which I have referred.

Those who are highly successful in mathematics are endowed

with certain mental qualifications that are of great use in many fields of intellectual effort. Among these qualifications may be mentioned, the power of concentration, the ability to generalize, the capacity for following a logical train of thought. No informed person would maintain that such abilities are of use only in mathematics. Those who have considerable difficulty with mathematics are usually deficient in some or all of the qualifications mentioned and would therefore find themselves handicapped in other fields than mathematics.

There has been too much of a tendency in the recent past to smooth the path of the high school student to an undesirable degree. In a sincere but sometimes misguided desire to encourage the student, efforts were made to make his course as easy for him as possible. In my opinion a student who is not willing to overcome a few difficulties does not deserve a high school education at public expense. And what is the value of education as a training for life if it does not teach us to surmount genuine obstacles? This is not a good reason for putting a subject into the curriculum merely because it is difficult, but it certainly is a good reason for not leaving it out merely because it is difficult. All the information at our disposal points to the fact that a knowledge of mathematics on the part of many citizens will be enormously helpful in extending the boundaries of our knowledge and enlarging our control of natural and social forces. The high school is obviously the place where the majority of our future citizens must obtain whatever serious training in mathematics beyond arithmetic they are apt to receive. We should therefore see that they get this training, both for the sake of society and the sake of the individual. We cannot do less and do our duty as educators.

The more progress physical sciences make, the more they tend to enter the domain of mathematics, which is a kind of centre to which they all converge. We may even judge of the degree of perfection to which a science has arrived by the facility with which it may be submitted to calculation.—Quetelet.

THE CULTURAL VALUE OF MATHEMATICS

By W. W. RANKIN

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Culture is an intelligent interest in the past, present, and future achievements of man. Surely there is no one who will say that culture is less than this.

A rather common notion of culture is that it has to do with literature only. But this idea is far too restricted to satisfy those who enjoy intellectual freedom and sympathy. Mathew Arnold's definition of culture is stated in this way: "Culture is knowing the best that has been thought and said." This definition of culture has too often been intrepreted in a very restricted way and this misconception has led to a more or less popular idea that the study of literature, and especially the classics is the broadest and most direct avenue to culture, and there are those who think it is the only avenue.

By an intelligent interest in a certain achievement we shall understand to mean, some knowledge of the history of the achievement, some knowledge of the underlying principles or laws which have assisted in making the achievement, and some knowledge of the value of the achievement.

Throughout this paper I shall use the definition of mathematics given by Benjamin Peirce who was for many years professor of mathematics at Harvard. Perhaps America has not produced a scholar who was more capable of defining mathematics. He said: "Mathematics is the science which draws necessary conclusions." Peirce had a very broad conception of the term science for he also said: "Mathematics belongs to every inquiry moral as well as physical. Even the rules of logic by which it is rigidly bound could not be deduced without its aid." It is only with this liberal view of the term science that we are able to classify mathematics as a science.

What is to be understood by "necessary conclusions" in this definition of mathematics? If you should arrive at a grade crossing at 11.55 A.M. and you should find a completely demolished automobile and a mangled corpse near by, and if you

knew that an express train had passed at 11.50 A.M., I believe it would be impossible for you not to draw a conclusion.

It is necessary to draw conclusions in order to make progress in our thinking. The evidence is summed up and a conclusion drawn, sometimes very quickly and at other times very slowly and carefully. *This process of weighing evidence and drawing the necessary conclusion is mathematics.* Compare this process with out phrase "jumping to conclusions" which by the way had at one time a good mathematical meaning, at least with Newton it did. He went out and jumped with the wind and then jumped against the wind and from this drew conclusions about the force of the wind. To-day this expression of "jumping to conclusions" is regarded as a sign of total absence of mathematics and is used as a reproach.

But where is the mathematics in the automobile accident? There is not much, the evidence was so complete and the conclusion so evident that the problem was quite simple. Just so is $2 \times 2 = 4$ in the number world a simple exercise. The process of drawing a conclusion is as fundamental in mathematics as is $2 \times 2 = 4$ in the number system.

Our textbooks and those of us who teach mathematics are so intent that our students shall acquire a certain amount of technique in manipulating the symbols used in mathematics that we have bounded mathematics on the North by x, on the East by $\sin A$, on the South by $\log x$, and on the West by $\sqrt{-1}$ in about the same way we learned to bound Ohio when we studied geography. The beauty and power of mathematics to set forth truth is lost sight of.

Professor Keyser of Columbia University says: "The domain of mathematics is the sole domain of certainty. There and there alone prevail the standards by which every hypothesis respecting the external universe and all observation and all experiment must be finally judged. It is the realm to which all speculation and thought must repair for chastening and sanitation, the court of last resort, I say it reverently, for all intellection whatsoever, whether of demon, or man, or deity. It is there that mind as mind attains its highest estate."

Compare this idea of mathematics with the idea that mathematics is a tool a notion common to many who study and to many who teach mathematics. The teacher who regards mathe-

matics as a tool might equally well think of himself as a chauffeur, showing the signs and wonders to his passengers.

There is nothing so inspiring as to feel a contact with something which is invariant among the many transformations which are daily taking place. The laws of mathematics afford us a touch with some of the eternal verities. Not even a college of Methodist Bishops can decree what the roots of a quadratic equation shall be.

Charles E. Hughes once said, "When the time comes that knowledge will not be sought for its own sake and men will not press forward simply in a desire of achievement, without hope of gain, to extend the limits of knowledge and information, then indeed will the race enter upon its decadence."

With these two definitions before us, namely: *culture is an intelligent interest in the past, present, and future achievements of man; and "Mathematics is the science which draws necessary conclusions,"* I want to study briefly with you the following list of achievements of man:

The Number System,	*Transportation and Communication,*
Language,	*Banking and Commerce,*
Process of Thinking,	*Medicine, and*
Art,	*Political Economy and Sociology.*
Architecture,	
Music,	

This list of achievements of man is far from complete but is a fairly comprehensive list.

The Number System

A moment's reflection will show how great an achievement the development of the Number System was for man. We need only to imagine that we have no number system to appreciate the value of this forward step.

Now let us ask ourselves seriously as men and women capable of reflection, what is an *intelligent interest* in the number system? The answer will come sharp and quickly from a large group, the utilitarians—the knowledge of how to *use* the number system. Another group perhaps smaller but with a wider range of interest will inquire; how, where, and when did man get a number system? What nations and individuals contributed

in this intellectual struggle? Is the system a complete one? What has become of Rithmomachia the great number game which in its day was the equal of our game chess? Tell us something about the origin and development of fractions, irrational numbers, imaginary numbers, the mysticism of number, and so on. A man has the opportunity to choose with which group he will answer—the utilitarians, or the students of civilization.

Language

Let us understand that language includes literature. Just how much mathematics goes into the composition of a language and the literature of that language, so far as I know has not been determined. Hudson Maxim says: "Sound which is the basis of language has four properties: *loudness, duration, pitch, tone color.*" All of these are physical properties and therefore capable of mathematical treatment. What is true here in regard to the mechanical side of language is also true in the content or thought if we can believe Thoreau who said: "The most distinct and beautiful statement of any truth must take at last the mathematical form. We might simplify the rules of moral philosophy as well as of arithmetic, so that one formula would express them both."

Mathematics preferably expresses its conclusions in the form of an equation, but may and often does use rhetoric. Before the days of symbols mathematics was expressed verbally. The Arabs and Hindoos even expressed their mathematics in verse. The usefulness of language would be greatly restricted if it were impossible to draw necessary conclusions.

R. W. Emmerson said: "We do not listen with the best regard to the verses of a man who is only a poet, nor to his problems if he is only an algebraist; but if a man is at once acquainted with the geometric foundation of things and with their festal splendor his poetry is exact and his arithmetic musical."

Process of Thinking

It is obvious that if "mathematics is the science which draws necessary conclusions," progress in thinking is closely associated with the development of mathematics.

Bertrand Russell the great mathematical philosopher said in

the International Monthly: "Pure mathematics was discovered by Boole in a work he called The Laws of Thought. His work was concerned with formal logic, and this is the same thing as mathematics."

I have already quoted to you what Peirce had to say on this point: "Even the rules of logic by which mathematics is rigidly bound could not be deduced without its aid."

The science of exact thinking in any field of thought is thus fundamentally dependent on the principles of mathematics, and the development of logical thinking cannot proceed faster than the development of mathematics.

Art

Art has to do with producing an effect on the observer of beauty of form, and beauty in color combinations. The highest art deals with beauty of forms. The Greeks recognized this, and have left to the world the best that has been produced in art. Those who have been careful to draw their conclusions from Nature where the Great Geometer has wrought such perfection in form and where the laws of proportion and symmetry obtain most perfectly, have been most successful in art. Nature furnishes also the basis for study in color combinations. Nature does not combine these colors by accident, for if she did we would have not one beautiful rainbow but factorial seven, or 5,040 rainbows.

Just how the Greeks attained such perfection in their art we are unable to say. Surely they knew the laws of symmetry and proportion and how to vary these laws in a most pleasing way. This very interesting suggestion has been made: that they knew the proportions of the members of the human body and that they worked this into their art; and that this pleases the eye because it finds harmony between the object observed and the various members of the body of which it is a part.

In 1844 D. R. Hay made a careful study of the Greek vases and their geometric proportions and also the proportions of the members of the human form. He was able with the composite ellipse, the ellipse with three foci, to produce the curved lines of vases which were quite similar to the curved lines of the Greek vases. He was also able with these composite ellipses of definite proportions to produce curves strikingly similar to the curves of the human form.

Architecture

Let us think of architecture in its broadest sense, including all structural design. Architecture might be thought of as the crystallization of human intelligence into geometric forms, or conclusions of the mind geometrized. It is quite beyond the scope of this paper to say how much mathematics has contributed to architecture. It would be very difficult to say just how much mathematics goes into the design of a cathedral and its construction, but far more than merely the laws of symmetry and proportion. In fact you can hardly think of anything in connection with the cathedral which has not first been subjected to the laws of mathematics. The skill of the architect lies in his ability to design and construct the cathedral in such a way that the observer sees the whole as one unit and not doors and windows and curves here and there, in other words his skill lies in his ability to use mathematics and then not have it visible when he has finished. Arithmetic, algebra and geometry must be in the cathedral but we must not see them with the eye, we must rather feel them with the intellect.

Music

Pythagoras who lived about 500 B.C. is regarded as the inventor of music as a science. He regarded music as applied numbers. Few people realize that music has a vital and necessary relation to mathematics.

Perhaps the most, or certainly one of the most interesting problems which occupied the mathematicians through the 18th century was that of the vibrating string and allied problems. These studies were carried on by Laplace and men of like ability, and after the invention of calculus the most powerful of al intellectual processes to analyze physical phenomena. It is from this research that we get our modern music. Out of this also grew Fourier's discovery of the method for expanding a function into a trigonometric series.

No one is better qualified to speak on music and mathematics than the great scientist and musician Helmholtz who has this to say: "Mathematics and music the most sharply contrasted fields of scientific activity which can be found, and yet related, and supporting each other, as if to show forth the secret connection which ties together all the activities of our mind, and which

lead us to surmise that the manipulation of the artist's genius are but the unconscious expression of a mysteriously acting rationality."

Leibniz, one of the greatest mathematicians of all times remarked: "Music is a hidden exercise in arithmetic of a mind unconscious of dealing with numbers."

Sylvester, the great English mathematician, asked this far-reaching question: "May not Music be described as the Mathematic of Sense, Mathematic as the Music of Reason? the soul of each the same? Thus the musician *feels* Mathematics, the mathematician *thinks* Music—Music the dream, Mathematic the working life—each to receive its consummation from the other when the human intelligence, elevated to the perfect type, shall shine forth glorified in some future Mozart-Dirichleet or Beethoven-Gauss—a union already not indistinctly foreshadowed in the genius and labors of a Helmholtz."

Of course there is no claim made here that it is necessary to study mathematics in order to study music. But the musician who knows the scientific basis of music has a more intelligent interest in this achievement of man than the musician who does not know the physical laws of sound. An intelligent interest demands more than a mere mastery of technique.

Transportation and Communication

Our modern systems of Transportation and Communication have called on the mathematicians for their conclusions more heavily than any other achievement of man. Man's conquest of nature in these two fields is due to a large extent to mathematics. The machinery for carrying on our transportation and communication is but a monument to the mathematicians. There are comparatively few who can say that they have an intelligent interest in the airplane, if an intelligent interest includes the mathematics of it. Wherever we find change the mind of man immediately raises the question of the rate of change, which is fundamentally a calculus idea.

It would be an interesting experiment to stop a Pierce Arrow on the road some day and weigh the brains riding in the car, and then weigh the brains which went into the design and construction of the car, if such an experiment were possible.

I should have to cause almost the whole of mathematics to

pass before you in order to show the part it has played in the development of transportation and communication.

Banking and Commerce

Commercial Mathematics is rapidly becoming a field of wide research. As the laws of trade relations are more thoroughly understood, they are reduced to formulas. The day has passed forever when the world's finances can be carried on with arithmetic. Business is rapidly becoming a science, but it can do this only in so far as it is capable of using mathematics. The one item of "Bond Issue" calls for daily assistance from the mathematician. The old professor of economics must now take up the study of calculus if he wishes to follow modern business as a science.

Medicine

It is very noticeable in the history of mathematics how many individuals were interested in medicine and mathematics; Cardan, Leonardo de Vinci, Robert Recorde (Royal Physician) who gave us our sign for equality (=), Lillio, physician to Gregory III (his suggestion was adopted for revising the calendar). Professor D. E. Smith gives a list of more than 100 in the sixteenth century alone who were interested in both mathematics and medicine. As we look back and inquire for a reason we find that it was largely due to their belief in astrology.

Modern medicine has been built up on the conclusions drawn in the laboratory where mathematics sits on the bench and hands down the decisions.

Dr. Alexis Carrel of the Rockefeller Institute of Medical Research has discovered that the surface of a healing wound may be reduced to a mathematical formula, involving the surface and the time of healing expressed in days. The formula is closely related to the formula for organic growth $y = e^{kt}$ and organic decay $y = e^{-kt}$. He is thus able to say how much of the surface of the wound should disappear in a given time, and if the healing is above or below the normal healing curve special investigation is necessary. The area of the wound is taken every four days by placing a thin paper over the wound and tracing lightly around the wound, this area is then obtained from the paper by means of a planimeter. The basis of the planimeter is calculus.

Geometry is essential to the study of optics, a fact that many people of more than average intelligence do not appreciate.

Few physicians to-day appreciate what mathematics has contributed to modern medicine; this is not necessary in order to be a good physician, but we are trying to think of him as a man of culture who knows the history of his subject, the underlying principles and laws of medicine, and that the value of medicine is proportional to its ability to discover and use the laws of man and nature in both a curative and preventative way.

Political Economy and Sociology

It is significant that colleges are offering courses under the general head of Social Sciences. Correctly interpreted, it means that in the political life and the social life relations are becoming sufficiently well understood to call them laws. In brief these relations in life are being reduced to a mathematical basis.

The most interesting book which has come from the press in recent years is called "Manhood of Humanity," The Science and Art of Human Engineering, by Count Alfred Korzybski. He gives a new definition of man, he defines man as a "time-binder" and distinguishes him from animal which he defines as a "space-binder." He claims that with this new definition man will be able to discover and reduce more of the moral, mental, and physical laws to a mathematical basis. We can only hope that he is correct.

Perhaps some future generation may have states rights and international relations so clearly stated that even a politician can understand them. I believe the greatest statement ever uttered was that of the Great Teacher, "ye shall know the truth and the truth shall make you free."

I see no difference in the sanctity of truth whether the truth be taught in the books of Euclid; in the holy books of the East, or in the Christian Church, in the significance of truth—yes; in the sanctity of truth—no.—Religio Mathematici, by David Eugene Smith.

MATHEMATICS PLUS

By FLORENCE BROOKS MILLER

Fairmount Junior High School, Cleveland, Ohio

On the editorial page of a recent newspaper I found a short article called, "The Drama of the Budget," which I should like to reproduce. It runs as follows:

THE DRAMA OF THE BUDGET

The budget of the United States may interest expert accountants, statisticians and others who get a great deal of enjoyment out of subjects of this character, but to the average citizen, who is unable to find in the great masses of figures any explanation of his tax bill, the budget provides little in the way of an evening's entertainment. And yet the semi-annual radiocasting of the proceedings of the business organization of the United States undoubtedly attracts a large audience.

Brig.-Gen. Herbert M. Lord, as Director of the Bureau of the Budget, has the happy faculty of being able to make a very dry subject something more than a mere rehearsal of facts and figures. In his hands the presentation of the budget rises almost to the proportions of a well-rounded theatrical performance. The United States Army Band furnishes the overture and incidental music and the President of the United States, rising amid the strains of "Hail to the Chief," the prologue.

Then comes the play in which the cast consists of that very versatile artist, General Lord, who not only instructs and educates but also amuses and entertains. Dusty statistics are made to dance about like minstrels while prosaic facts assume attractive garb and are accepted cheerfully, or at least with a wry smile, by even those department heads which find in them the instrument for cutting their appropriations. And through it all General Lord tells stories, tells them with geniality and effect, tells them well. The band plays, the curtain drops, the seen and unseen audiences applaud and The Budget, a one-act drama—tragedy or comedy, however you may look at it—comes to an end for another six months.

General Lord evidently appreciates the fact that in order to have people listen and learn, there must be present that which holds the attention, which entertains, being at the same time substantially worthwhile.

In his book *Curriculum Problems*, Dr. Thomas H. Briggs has a chapter on "Emotionalized Attitudes," in which we find such statements as "We feel more than we think." "Very few actions have immediate intellectual causes unaffected by feel-

ings." An unknown psychologist has written, "Our intellect is a mere speck afloat on a sea of feeling."

Referring again to the newspaper article may I repeat a striking sentence. "General Lord not only instructs and educates but also amuses and entertains." Undoubtedly without the amusement and entertainment, the instruction and education would have been only for a very few people, namely, those who already had had their interest in the subject aroused and had needed no extra inducement to hold their attention.

Have we any control over this sea of feeling? Dr. Briggs makes this important statement: "The school is teaching attitudes whether we will or not—not only teaching attitudes but also, which may be of far more importance, suffusing them with feelings which, varyingly persistent, insure definite actions." If a pupil dislikes a subject, he is not likely to continue studying it when it is no longer compulsory. Thorndike says, "Every experience one has is a satisfier or an annoyer." Briggs continues, "Each satisfier and annoyer carries with it a feeling or emotion. It is that which determines whether we want more of it or not."

We all recognize the importance of hobbies. People without hobbies are usually one-sided and narrow. Important discoveries for the good of mankind can come from someone's interest in a hobby. The study of hobbies made by Dr. Briggs is of interest to us teachers. Of 2,000 papers which stated interests, the median age of the beginning of the interests was between ten and fourteen, the junior high school age, and only three percent could be traced to school work. The interests happened. The origin was usually personal contagion. I have a feeling that ten years hence a similar study would reveal the fact that many interests will have been aroused which could be traced directly to school work.

The junior high school, which recognizes that one of its most important functions is to provide for exploring in various fields of activity, introduces many boys and girls to the beginnings of interests which will become worthwhile hobbies if not their life work. A thirteen-year-old boy joined one of the mathematics clubs in our school. The manipulation of the slide-rule was undertaken by this club. After they had learned to use it to multiply, divide, find powers and extract roots, the leader spent

a period explaining briefly the functions of the right triangle, showing that they are simple ratios that can be derived quickly by the use of the slide-rule. Following this several periods were spent discussing the various devices that are used in finding distances by indirect measurement. Several days later this boy showed to the leader of the club a gift from his father, a slide-rule which cost ten dollars. He was proud of the rule but his primary object in bringing it was to receive help in the use of it in the solution of a formula in which was involved the trigonometry of the general triangle. By the use of the handbook which came with the slide-rule, he had already mastered the solutions of the formulas of the right triangle. This same boy has constructed a miniature transit, and is looking forward with great interest for good weather when he will be given an opportunity to use a real transit which is the property of the school.

Before clubs were generally organized in our school, a request came from some of the members of an algebra class to form a club so that they might have an opportunity to learn more about some of the historical bits regarding the formation of number systems, origin of symbols, stories about early mathematicians, and so on. It was not uncommon to have members of that class ask for some of the class time to give a mathematical puzzle or to explain a peculiar method of multiplying, one having to know only the table of twos, and so on. Needless to say the club was formed and was a very lively and worthwhile one.

Those of us who are familiar with the teaching of intuitive geometry realize with what interest the pupils make posters which illustrate geometric forms and ideas. Showing samples to them is usually all that is necessary to make them want to do something similar. We have an excellent example of what can be accomplished by understanding human nature as well as mathematics in an article by Miss Glenna Wells of Thomas Jefferson Junior High School, Cleveland, which appeared in the January number of *The High School Teacher*. The article tells how Miss Wells had on her desk a very attractive array of books on architecture. These books were handed out as a favor to those pupils who had completed an assignment. Thus they became very desirable. In fact such a splendid foundation for a study of architecture was so made that hours were spent by pupils together with the teacher in the Public Library and in

the Art Museum on Saturdays. Another hobby, if not a life's calling, has perhaps thus been instigated for more than one child. Surely this is an excellent example of the cultural value of mathematics also.

It has been my good fortune to have come into possession of some mathematical books which date back to 1849 and 1864. In reading the prefaces in these old books, one senses something very familiar, some sentences which might well be found in a book just off the press. For example we find such statements as these: "Teachers now recognize the natural as the logical order of subjects, and the order of simplicity as best adapted to the natural unfolding and growth of the mind. Pupils are taught to reason from cause to effect rather than to follow the blind direction of rules, to walk unaided rather than to be carried like deformed cripples." In the same preface we find a sentence which betrays its age. It is this: "In this work an effort has been made to furnish the pupil with ample material systematically graded, to give that discipline to the mind which will fit him for the practical duties of life."

Notwithstanding the very promising statements in the first quotation above, an examination of the book proves quite disappointing, the general plan throughout being: definition, model problem, solution of model problem, followed by many similar problems to be worked in the same way as the one given.

In *Modern Methods in High School Teaching*, by Douglas, p. 72, we read: "Teachers have fallen largely into the error of regarding their task as the traversing of certain traditional units of subject-matter, rather than as the task of guiding and promoting the development of boys and girls along desired lines. Because the process of development is long continued, and is accomplished only through daily use of small items of subject-matter, it has been easy to concentrate upon these daily tasks to such an extent that perspective is lost, and the purposes for which the daily tasks are done slip out of the focus of attention."

No doubt for a long period of time, the ultimate test of the value of our teaching will be judged by the ability pupils have acquired in the use of essential mathematical processes. The successful teaching of these, however, is dependent, to a larger degree than is often realized, upon those factors which we are apt to consider of only incidental importance. Before the pupil

can be brought to accept willingly the task of learning any one of these processes, he must have the aid of proper emotionalized attitudes in order to accept the task as worthy of his effort.

The Project Method and Socialized Recitation are in answer to a crying need to make the subject-matter more appealing and more real to the pupils. Unless handled with great thought and care, these methods are wasteful and have been known to fail in their purpose because of the lack of time devoted to drill which is essential. Sometimes the project is continued after its usefulness is over.

To try to make real to children the less intricate situations connected with buying and selling, I have used the following plan:

In our minds we go to a large department store. We speak of the things to be seen there which have made an impression upon us. The many things to be sold, the glass show-cases, the "dressed-up" models in the windows, the lighting, the warmth of the store, carpets on the floor of some departments, the many clerks, and so on, have been mentioned by members of the class. One boy mentioned the sprinkling device on the ceiling for extinguishing fire; a girl spoke of the moving-stairway and elevators.

Where do the things which are for sale come from? That question has brought forth replies such as "the warehouse," "the wholesale house," "a factory." Each answer is met by a question such as, "And where does the manufacturer get the material?" In this way the meaning of sources is cleared up and sheep are spoken of in connection with woolen things; silk-worms, with silk; forests with furniture; mines with articles of iron, steel, gold, silver, and coal.

The class is then told that we are to have for our use the cork-board ($8' \times 4'$) in the back of the room, on which to place pictures which will show the important steps taken in order to have things reach our homes from their original sources.

This term I am trying this plan with the slowest group of 7A's in our building. They have become very much interested and have brought in a surprising array of pictures which are placed in columns with the following captions:

BUYING AND SELLING

SOURCES MANUFACTURING WHOLESALE HOUSE RETAIL STORE HOME

Between every two columns are pictures of means of transportation, such as an airplane, a train, a boat, a truck. Bright colored yarn is used, a different color for each article, to trace its journey from its source to the manufacturing establishment, to the wholesale house, then to the retail store, and finally to a room in the home.

The height of interest is reached when the whole class, thirty-nine in number, are gathered around the bulletin-board, each one given an opportunity to make a suggestion if he wants to, as the yarn is fastened by thumb-tacks to the pictures.

Two full periods have been devoted to this kind of work. This enabled us to have on display enough to motivate problems that involve costs, overhead expenses, gross cost, selling price, gross gain, loss, net gain, rate of gain or of loss when based upon the cost or the selling price. Each day now some time is given to a little work on the group of illustrations, adding new pictures, tracing a product from source to home, and talking over the overhead expense which must be met by each one who handles the article. The net gain we think of as the payment for the accommodation.

The group of pictures serves well also in motivating commercial discount, the meaning of list price or marked price, the reasons for reductions, and so on. In fact it may be used in the teaching of commission. Salesmen working on a commission basis can be imagined going from the wholesale to the retail stores. Buyers, too, may be traced as going even to some of the sources.

The hope is that aside from learning the mere mathematical essentials of buying and selling, the boys and girls will acquire an understanding of the conditions which naturally produce certain problems. It is not unusual to have questions of fairness, honesty, promptness in making payments, as well as in meeting appointments, arise in the class discussions. These are some of the things I mean by "Mathematics Plus." If interest may be considered as an important criterion by which to judge learning, these pupils are learning because the interest is unmistakable, largely due to "Mathematics Plus."

Nothing which I have tried has brought home to the pupils

a good understanding of commercial discount, as well as the use of advertisements from the daily newspaper. When the class is asked to bring in newspaper advertisements concerning reductions in the price of articles, they are asked also to find out the cost of advertising. When it is known that full-page advertisements cost between $350 and $700 a day, depending upon the contract the company, which is advertising, has with the newspaper, many are the remarks; keen is the interest. One little girl said, "Yet it must pay to advertise because they keep on doing it."

An informational lesson on causes for reductions prepares for the study of the advertisements which the pupils have brought in. Each pupil makes at least one original problem based upon his advertisement which is pasted upon the paper with the problem. The work of the problem is also shown on the sheet. Such exercises necessitate an understanding of list or marked price, discount, sale price, rate of discount.

A booklet compiled by Miss Martha Cook,[1] called "Percentage Applied," is very suggestive and worthwhile. It is a writeup of class recitations, not stenographic.

We cannot get away from the necessity of arousing interest in that which we teach. To do this a teacher must keep in mind that attitudes are being taught if nothing else. If the right attitudes are being acquired, no doubt the subject-matter is too. In order to create enthusiasm a teacher must herself be enthusiastic. In order to be enthusiastic she must keep getting fresh funds of information and well-thought-out methods of presentation. What a wealth of helpful material we have easy access to at the present time! What a wonderful contribution has been given to us by one man, Dr. David Eugene Smith, of Teachers College, Columbia University. His subjects are profound yet written in such a way as to interest and make one desire more. His history of mathematics is a gold mine of authentic information. Having read and re-read Dr. Smith's article "Mathematics in the Training for Citizenship,"[2] a teacher should never be at a loss to reply to anyone who might ask, "Why should I study mathematics?" "What is there in that dry subject, math-

[1] Published by The Harter School Supply Co., 2046 E. 71st St., Cleveland, Ohio.

[2] See *Third Yearbook of The National Council of Teachers of Mathematics*, page 11.

ematics, that makes you want to teach it?" In this paper Dr. Smith makes this plea: "Let us see to it that the poetic side of mathematics is recognized as well as the practical side; let us see to it that we show the world how to use its leisure as well as how to turn the restless wheels of industry." Not only is this plea made, but Dr. Smith gives enough information to assist one along this line. Fortunately, there are other articles by this same scholar which have been appearing in THE MATHEMATICS TEACHER from time to time. In the December 1927 number we find "Esthetics and Mathematics"; in January 1928, "Introduction to the Infinite"; in February 1928, "The Form of the Universe"; in April 1928, "The Lesson on Dependence," an interesting treatment of the function concept; in May 1928, "Time in Relation to Mathematics."

The many splendid articles in *The Yearbooks of the National Council of Teachers of Mathematics* and in each number of THE MATHEMATICS TEACHER, descriptive lessons, historical bits of information, plays, discussions regarding ability grouping, are very worthwhile. The articles which in my estimation are most helpful, are those which open our minds to whatever will cause us to see the opportunities constantly arising in our classes, to create desirable attitudes, and to correct those which are not desirable.

"Mathematics Plus," that very essential promoter of interest, understanding and retention, depends for its existence upon the teacher who has a thorough understanding of the subject matter, a growing knowledge of interesting bits of mathematical history, a fund of general information upon which to draw for illustrations, and contagious enthusiasm.

On this Occasion, I must take notice to such of my readers as are well vers'd in Vulgar Arithmetik, that it would not be difficult for them to make themselves Masters, not only of all the Practical Rules in this Book, but also of more useful Discoveries, if they would take the small Pains of being acquainted with the bare Notation of Algebra, which might be done in the hundredth part of the Time that is spent in learning to read Short-hand. —The Doctrine of Chance, Preface to First Edition (1718), vii.

THE TENTH ANNUAL MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, CLEVELAND, OHIO, FEBRUARY 22, 23, 1929

MINUTES OF THE JOINT MEETING OF THE BOARD OF DIRECTORS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS AND THE CLEVELAND LOCAL COMMITTEE, HELD IN THE HOTEL STATLER, CLEVELAND, OHIO, FRIDAY MORNING, FEBRUARY 22, 1929

Present from Board: Marie Gugle, Vera Sanford, Mary S. Sabin, Harry C. Barber, C. M. Austin, J. O. Hassler, William Betz, Harry English, Edwin W. Schreiber and W. D. Reeve.
Present from Local Committee: D. W. Werremeyer, A. B. Miller, Fred N. Burroughs.

President Barber called the meeting to order at 10:10 A.M. Mr. J. A. Foberg, Secretary-Treasurer, was unable to attend the meeting, due to serious illness. Edwin W. Schreiber of Ann Arbor, Michigan, was made Secretary-Treasurer, pro-tem.

Fred N. Burroughs, President of the Cleveland Mathematics Club made a report concerning the work done by the Local Committee in the way of advertising the meetings and arranging accommodations at the Hotel Statler.

President Barber then presented the matter of the *Affiliation of the National Council of Teachers of Mathematics with the Mathematical Association of America.* It reads as follows:

It is agreed

1. *That members of the National Council of Teachers of Mathematics who meet the qualifications for membership in the Mathematical Association of America may become members of the Association without the payment of the customary initiation fee, the National Council to supply to the M. A. A. from time to time a list of new members whom the M. A. A. may invite to their membership under this special condition.*

2. *That it is understood that whenever the M. A. A. or any of its Sections holds meetings in a location where a Branch of*

the National Council exists, the Branch engages to aid in such meetings, and that meetings of the National Council or its Branches as an affiliated organization will be welcomed in cooperation with the meetings of the M. A. A.

3. *That the present agreement may be terminated by either party on six months' notice.*

4. *That a vote of approval by the Trustees of the M. A. A. and the Directors of the National Council shall make this agreement effective.*

Approved by the Trustees of the M. A. A. Dec. 28, 1928.

(Signed) W. D. CAIRNS,
Secy.-Treas.

Approved by the Board of Directors of the National Council of Teachers of Mathematics, Feb. 22, 1929.

(Signed) EDWIN W. SCHREIBER,
Secy.-Treas.

On motion of W. D. Reeve, seconded by Wm. Betz, the Board of Directors of the National Council approved the proposed affiliation with the Mathematical Association of America.

Professor Dunham Jackson of the University of Minnesota announced the existence of a committee of five, appointed by the M. A. A., to investigate the feasibility of a one-year course in geometry for our secondary schools, said course to contain the elements of plane and solid geometry. He desired the cooperation of the National Council in aiding the committee and suggested that several members of the National Council be placed on the committee to make it a more representative cross-section of the mathematical interests of teachers of geometry. Professor Ralph Beatley, of Harvard University also spoke with reference to this Committee on Geometry.

After some discussion it was moved by J. O. Hassler, seconded by C. M. Austin, that the National Council cooperate with the M. A. A. in the appointment of a committee on geometry and that the National Council share the expense up to $200, provided that the five members representing the National Council be classroom teachers of geometry in public high schools. Carried.

It was moved by Marie Gugle, seconded by Harry English, that a new five-year contract be drawn up with W. D. Reeve as Editor of the MATHEMATICS TEACHER. Carried.

Professor W. D. Reeve made a brief report concerning the MATHEMATICS TEACHER and the Fourth Yearbook.

It was moved by W. D. Reeve, seconded by C. M. Austin, that the Secretary be instructed to send a letter of appreciation to Professor W. F. Osgood and his committee for their aid in securing contributors to the Fourth Yearbook. Carried.

The joint meeting of the Board and Local Committee then adjourned.

MINUTES OF THE ANNUAL BUSINESS MEETING, FRIDAY, FEBRUARY 22, AT 2:30 P.M.

President Barber called the meeting to order and called for the reading of the minutes of the Boston meeting, 1928. Edwin W. Schreiber, Secretary-Treasurer pro-tem, read extracts from the minutes as compiled by J. A. Foberg in the "official corporate records." A brief report on the condition of the treasury was also made.

Miss Vera Sanford reported for the Auditing Committee that the records were found correct. Balance on hand February 12, 1928—$2,246.15. Received during the year 1928—$849.71. Balance on hand February 7, 1929—$2,396.45.

W. D. Reeve reported on the present status of the MATHEMATICS TEACHER, and also on the Fourth Yearbook. A motion was made and unanimously carried to extend a vote of thanks to Professor Reeve.

Professor W. W. Rankin of Duke University proposed the compiling of a source book for elementary mathematics. On motion the matter was referred to the Board of Directors.

President Barber then made his annual report which gave every evidence that our President has been a very busy man making arrangements for the Cleveland Meeting and taking care of a great amount of correspondence.

The Annual Business Meeting was concluded by the announcement to the members at large of two important matters approved by the Board of Directors: (1) Affiliation with the Mathematical Association of America; (2) The Committee of Ten on Geometry, five representing the M. A. A. and five the National Council. The members of the Committee are:

Professor Dunham Jackson, University of Minnesota, Chairman
Professor C. N. Moore, University of Cincinnati

Professor J. O. Hassler, University of Oklahoma
Professor W. D. Reeve, Teachers College, Columbia University
Professor Ralph Beatley, Harvard University
Gertrude E. Allen, Oakland, California
C. M. Austin, Oak Park, Illinois
Walter F. Downey, Boston, Massachusetts
Elizabeth L. Hall, Rochester, New York
Edwin W. Schreiber, Ann Arbor, Michigan.

Open Meeting, Friday Evening, Ferbuary 22, 1929

About 250 were in attendance at the first open meeting of the Council. President Barber presided and introduced as the first speaker, Mr. C. H. Lake, Assistant Superintendent of Schools, Cleveland, who gave the Address of Welcome.

Fred N. Burroughs, President, Cleveland Mathematics Club, brought greetings from the Local Organization.

Mary S. Sabin, Second Vice-President of the Council, came all the way from Denver, Colorado, to make the response for the Council.

The address of the evening was made by Professor Louis C. Karpinski of the University of Michigan, who spoke on, "The Permanent Nature of the Necessity for Mathematics in the Secondary Schools." This address is published in full in this issue.

Open Meeting, Saturday Morning, February 23, 1929

The following program was presented:

9:00 A.M. Open Meeting .... Lattice Room

Current Tendencies in Mathematics Teaching.

A World-Wide Survey Based on the Reports in the Fourth Yearbook .... Vera Sanford

The Lincoln School, Teachers College, Columbia

Mathematics and the Future .... Charles N. Moore

University of Cincinnati

Present Trends in Curriculum Reconstruction .... William Betz

Rochester, New York

For the Good of the Council .... W. D. Reeve

Teachers College, Columbia

Discussion

Open Meeting, Saturday Afternoon, February 23, 1929

The following program was presented:

2:00 P.M. Open Meeting .... Lattice Room

Mathematics Plus .... Mrs. Florence Brooks Miller
Fairmount Junior High School, Cleveland

The Cultural Value of Mathematics .... W. W. Rankin
Duke University, Durham, North Carolina

The Place of Mathematics in Junior High School Education .... Agnes Grant Rowlands
Jamaica Training School and Hunter College, New York City

Teaching Geometry into Its Rightful Place .... J. O. Hassler

The following new officers were declared elected:

Second Vice-President, Hallie S. Poole.

Members of Board of Directors: John R. Clark, Elizabeth Dice, J. O. Hassler.

Minutes of the Meeting of Board of Directors, Saturday Noon, February 23, 1929

It was moved by C. M. Austin, seconded by William Betz, that Vera Sanford be made Assistant Editor of the Mathematics Teacher. Carried.

It was moved by Marie Gugle, seconded by C. M. Austin, that Edwin W. Schreiber be made Secretary-Treasurer of the National Council for an indeterminate term. Carried.

It was moved by Marie Gugle, seconded by Harry English, that President Barber send a letter to J. A. Foberg expressing our sincere sympathy in his serious illness, and, furthermore, to make known to him our deep appreciation of his nine years of service as Secretary-Treasurer of the National Council. Carried.

It was moved by J. O. Hassler, seconded by Marie Gugle, that Mary S. Sabin fill the unexpired term of Edwin W. Schreiber on the Board of Directors (1931). Carried.

It was moved by Edwin W. Schreiber, seconded by Harry English, that the Secretary-Treasurer be instructed to have an appropriate Certificate of Affiliation designed to send to the various Branches as evidence of their affiliation. Carried.

It was moved by J. O. Hassler, seconded by C. M. Austin, to request the Editors of The Mathematics Teacher to establish in

our journal a Department for "Official Notes" or "Council Notes." Carried.

It was moved by Edwin W. Schreiber, seconded by J. O. Hassler, that an Index be compiled for the MATHEMATICS TEACHER for the period 1921–1928 and if necessary the expense to be borne by the Council. Carried.

It was moved by Marie Gugle, seconded by Harry English, that from 1929 on, each volume of the MATHEMATICS TEACHER contain a yearly index equivalent to that in the *Mathematical Monthly*. Carried.

It was moved by C. M. Austin, seconded by J. O. Hassler, that as soon as the place of the Annual Meeting has been determined a notice of the said meeting be printed in each issue of the MATHEMATICS TEACHER and a tentative program of the meeting be published in the January issue. Carried.

The meeting then adjourned.

THE ANNUAL BANQUET, SATURDAY EVENING, FEBRUARY 23, 1929

Shortly after six o'clock, 165 members and guests sat down to a delicious banquet served in the Lattice Room of the Statler Hotel. The head table was beautifully decorated with flowers and place-cards and a big birthday cake adorned with ten candles. President Barber presided as toastmaster and contributed much to the general mirth and good fellowship of the occasion. He called upon C. M. Austin, our first President, to cut the birthday cake and tell of the first years of the Council. Professor C. W. Cairns, Secretary-Treasurer of the M. A. A., brought the greetings from his association. Principal F. P. Whitney brought greetings from the local city. Many other guests of honor responded to toasts on the call of the tostmaster. Then came the real treat of the evening, a splendid address by Professor Charles H. Judd, University of Chicago, on "Informational Mathematics versus Computational Mathematics." We were greatly honored in having Professor Judd as our guest. His address appears in this issue.

EDWIN W. SCHREIBER,
Secretary-Treasurer.

NEWS NOTES

The following schools have reported 100 percent membership in The National Council:

Tallahatchie County Agricultural High School, Charleston, Missippippi.

North High School, Omaha, Nebraska.

Cleveland Intermediate School, Detroit, Michigan.

Professor W. H. Sherk of the Mathematics Department of The University of Buffalo died recently at his home at Buffalo. He was a member of The National Council and The Mathematical Association of America and last year was president of the Association of Teachers of Mathematics in the Middle States and Maryland.

Members of The National Council will be pleased to learn that The Third Yearbook of The National Council of Teachers of Mathematics on *Selected Topics in Teaching Mathematics* was selected as one of the *Sixty Educational Books* of 1928 by the Journal of The National Education Association.

The Mathematics Section (Section 19) of the New York Society for The Experimental Study of Education held its fifth dinner meeting of the current year at The Men's Faculty Club of Columbia University on Saturday, March 2, at 6.30 P.M. Dr. J. Carleton Bell of the College of The City of New York was the guest of the evening and represented the New York Society with some remarks about the organization. Dr. W. S. Slauch of the High School of Commerce reported on his experiences at the Cleveland meeting and Dr. Vera Sanford of The Lincoln School presented The Fourth Yearbook of The National Council of Teachers of Mathematics.

This section meets regularly once each month and for the past two years has had an average attendance of 85.

The Philadelphia Section of The Association of Teachers of Mathematics of the Middle States and Maryland has held two meetings this year. The first was a dinner meeting on Friday evening, November 23, at Houston Hall, University of Pennsylvania. The guest of honor was Mr. Harry C. Barber, president of the National Council of Teachers of Mathematics, who spoke on "Some Aims and Criticisms of Secondary Mathematics Teaching and How We May Hope to Meet Them." Brief speeches appropriate for the occasion were made by Dr. Wheeler, Dr. Wilson, Dr. Durell and Mr. Halman White.

The second meeting was held at Central High School on Saturday morning, March 9, 1929, with Dr. Clarence Garbrick presiding. Professor W. D. Reeve of Teachers College, Columbia University, spoke on "Mathematics and The Imagination." The new officers for the coming year are as follows:

President—Dr. Perry A. Cario, University of Pennsylvania.

Vice-President—Amy L. Clapp, South Philadelphia High School for Girls.

Secretary-Treasurer—Mary L. Constable, Girls High School.

The executive committee of the Association of Teachers of Mathematics of The Middle States and Maryland held a meeting at Philadelphia on March 9, 1929. It was decided at that time not to hold a Spring meeting of the Association due to the numerous meetings being held by many groups in the same territory.

Inasmuch as there seems to be some question as to the future of the organization due to the lack of interest shown by some of the former sections of the Association a committee was appointed by the president to consider the matter of reorganization. The committee was ordered to report at the Fall meeting in 1929. Dr. J. T. Rorer of the Central High School, Philadelphia, is chairman of the committee.

THE CLEVELAND MEETING

General Theme

The annual meeting of the National Council of Teachers of Mathematics was held at the Hotel Statler in Cleveland, Ohio, February 22d and 23d. The general theme for discussion was *The Place of Mathematics in Education.* It was the purpose of the meeting to emphasize again to the teachers of Mathematics and to educators generally the rapidly growing importance of mathematics and the unique position that it has among school subjects. Dr. H. E. Slaught of Chicago says: "Mathematics underlies our present day civilization in much the same fundamental way as sunshine forms the source of all life and activity on the earth." We are living in a universe where there is civilization and intelligence; and where there is intelligence there is mathematics.

This is a scientific age, and all scientific study is based on numerical data. It is also an industrial age in which the increased production that has taken place in the last few years is enormous. There have been increased demands for knowledge of mathematics in surgery, commerce, trade, the weather bureau, and modern invention. One field after another is being compelled to turn to mathematics for the basic principles which are so essential.

Cleveland Hospitality

The Cleveland Mathematics Club was a host worthy of the name. A committee headed by their president, Fred N. Burroughs including D. W. Werremeyer, J. W. Jacobs, A. Brown Miller, C. B. Temper and Elizabeth Thomas relieved the Council officers of many details.

The setting aside of one room, by the management of the Statler, for all the sessions of the Council made for prompt and regular attendance of the members.

Growth of Council

Several mathematics clubs asked for affiliation with The National Council on this its tenth birthday. Those present who have piloted the organization since its birth had every reason to be proud of its de-

velopment and growth—and to anticipate a future stronger in every way.

Mathematical Geography

The curves of interest, profit, and enjoyment reached their maxima at a common point—the banquet room. Those present were asked to stand in response to calls by cities or states or sections. Starting with cities these were the principal delegations: Cleveland 42, Detroit 19, Columbus 18, Chicago 10, Pittsburgh 7, Rochester 6, New York 5, Buffalo 5.

Four states were called: Ohio, outside of cities named, 13; Michigan, outside of Detroit, 7; New York, outside of cities called, 3; Pennsylvania, outside of Pittsburgh, 1.

The remaining parts of the country were divided into five sections, representing as follows: New England 2, other Atlantic States 1, Southern (south of Ohio and Missouri Rivers) 4, North Central 10, Western and Pacific 3.

In this enumeration, several mistakes were unavoidable. Accordingly the total number present was not 156 as indicated but about 175. The fine spirit of the whole meeting and the particular message of Dr. Judd will be carried to all parts of the United States.

The outstanding features of the entire session were the keen interest and attentiveness of the entire group throughout the series of meetings and the fine spirit of good fellowship.

GRACE SHICK,
WARD I. LYONS,
ARNOLD V. DOUB,
RUTH H. UTLEY.

On January 18th, 1929, the Chicago Men's Mathematics Club voted unanimously to affiliate themselves with the National Council of Teachers of Mathematics. This increases the list of branches or affiliated Clubs to eight according to the records now available. The Clubs and organizations so far reported are as follows:

1. Alpha Mu Omega, State Normal School and Teachers College, Peru, Nebraska. Detroit Mathematics Club.
2. Association of Teachers of Mathematics in the Middle States and Maryland.
3. Columbus (Ohio) Mathematics Club.
4. Connecticut Valley Section, Association of Teachers of Mathematics of New England.
5. Louisiana-Mississippi branch of the National Council.
6. Mathematics Section (19) of the New York Society for the Experimental Study of Education.
7. Mathematics Section, Oklahoma State Educational Association.

If any clubs or organizations have been omitted THE MATHEMATICS TEACHER would like to be notified.

The Kansas Association of Mathematics Teachers was organized April 16, 1904 and has been active ever since that date. At present it has a membership of 112 members. The heads of the five Mathematics Round Tables, the president, the vice-president, the editor of the *Bulletin*, and the secretary-treasurer form an executive board for the association. Our official meeting time is the first Saturday in February but we keep in close touch with the Round Tables which meet in

the fall and take in many members at that time. Besides recommending textbooks for state adoption, helping to reorganize the state courses of study, and the like, we are now publishing a quarterly, *The Bulletin*, which we send to each of our members. Although we have published only six issues we find that it is accomplishing its purpose in spreading helpful suggestions on methods and materials to be used in teaching secondary mathematics and in giving our members necessary information about the association and its meetings.

Our last meeting was held February 2, 1929, in Topeka, Kansas. The Kansas Section of The Mathematical Association of America met with us in the morning session. In the afternoon each held its own meeting. The following program was given:

PROGRAM

Miss M. Bird Weimar, presiding

"The Value of Mathematical History to the Teacher and the Pupil," Prof. J. O. Hassler, University of Oklahoma.

Report of the Mathematical Congress at Bologna in 1928, Prof. E. B. Stouffer, University of Kansas.

General Discussion.

Business Session.

11:30 A.M.

Joint Luncheon followed by Toast.

"Mathematics and Poetry," Prof. U. G. Mitchell, University of Kansas.

1:00 P.M.

"Recent Influences in the Teaching of Geometry," Prof. J. O. Hassler.

2:00 P.M.

Session of the K. A. M. T.

"Teaching Geometry by Means of Guide Sheets," Miss Helen Houghton, Manhattan.

"Teaching Algebra to Bring out Fundamental Distinctions," Miss Ethel Rumney, Emporia.

"Methods in Teaching Elementary Algebra," Miss Edna E. Austin, Topeka.

2:00 P.M.

Session of the M. A. A.

"The Gamma Function," Prof. C. H. Ashton, University of Kansas.

"Some Properties of Euler's Phi Function," Prof. D. H. Rickert, Bethel College.

We were very glad to have a member of the Executive Board of the National Council, Mr. J. O. Hassler of the Unnversity of Oklahoma, with us since we are interested in organizing a Kansas branch of the Council. Sixty-three of our members have subscribed for THE MATHEMATICS TEACHER through the Association and others have subscribed individually. A motion was made and carried to let the Executive Board make the final decision in the matter. The Board approves, but has not yet worked out the details of the change.

CARMILLE HOLLEY,
Secretary-Treasurer K. A. M. T.

Professor C. H. Judd of the School of Education of the University of Chicago in speaking to the Cleveland Teachers' Federation recently said:

"School is the place where society is trying to perpetuate itself. You can not dictate all the activities of the future, but you may expect to train the man of the future so that he will formulate principles more effectively and forcefully than this generation has been able to do. Gifts of civilization are not static

gifts. You are giving the child devices of thinking with the expectation that he will broaden them.''

Dr. Judd explained why arithmetic and language are so hard for the child.

According to Dr. Judd, arithmetic is one of the most complicated inventions of society—a mechanical device it took ages to evolve. He pointed out the crude devices primitive peoples used in computing. He asked his audience to multiply in Roman numerals, which were the only numerals Europe had to the sixteenth century.

''Arabic numerals brought to Europe the precise determination that is capable of carrying the weight of thinking that makes civilization possible,'' he said.

''If it took so long to evolve, why expect it to be easy for the child to master?

''Besides we ask a child to do impossible things. We ask him, for instance, to add 0 to 4.

''He tries to please his teacher. He knows she expects him to do something. He knows what 'add' means. Hence he gets the wrong answer.''

The speaker declared that there are about 410 words which may be used instead of the plus sign, hence another difficulty for the child.

NEW BOOKS

The Calculus. Robert D. Carmichael and James H. Weaver. Ginn and Company. 1927. Pp. viii + 345. Price $2.80.

The authors have neither added nor eliminated any calculus content. They have not introduced ''the most powerful tool'' in a new or different way. They have not, in the main, either blended or rearranged the traditionally scattered subject matter.

Yet, the new textbook in the calculus is as deserving of publication as any of its predecessors. What makes it more than merely another textbook on the market is the excellence of an unusually refreshing style evinced by a clear (though not uniformly simple) mode of presentation.

Of surprisingly few minor blemishes one only will be pointed out: An otherwise effective first chapter is spoiled by ''Formulas for reference,'' which might better have adorned an appendix.

Joseph Seidlin.

Elementary Calculus. Frederick S. Woods and Frederick H. B. Bailey. Ginn and Company, Revised Edition, 1928. Pp. x + 385. Price $3.00.

This popular and well-known textbook has had most of its minor faults removed, and none added, in the revised edition. The new material in analytic geometry will undoubtedly please the older users of the book and may gain some new ones. The most important improvement however, is in the early, natural, and consistent development of integration. In fact, the authors have almost achieved a ''unified'' calculus.

Joseph Seidlin.

Arithmetik für die I.–III. Klasse. Erwin Dintzl. 2. Auflage, Hölder-Pichler-Tempsky, Vienna, 1928. Pp. v + 244 + tables.

Arithmetik für die IV. Klasse. Erwin Dintzl. *Ibid.*, 1927. Pp. iv + 141.

Geometrie für die I.–III. Klasse. Erwin Dintzl. 2. Auflage. *Ibid.*, 1928. Pp. vi + 201.

These three books are based upon the works of Močnik and Hočevar, long and favorably known in the schools of Austria. They bring the latter into conformity with the decrees of the educational ministry of 1926 and 1927, and hence are representative of the best work in the Mittelschulen at the present time. In comparing this work with what is done in our American schools, the following will be of service:

Mittelschule	American Grades	Age of Pupils
I	........5th	11 yr.
II	........6th	12 yr.
III	........7th	13 yr.
IV	........8th	14 yr.

With this basis of comparison, the following statement may be of interest to our teachers:

Classes I–III. The work in the writing of numbers (Hindu-Arabic and Roman) is more formal than with us, and the Roman numerals conform to local usage rather than to historic precedents. For example, MCM for 1900 is local there and here, but is not the common historical form. Paper cutting and folding are used to explain the significance of units, tens, and so on. The terms *Summanden, Posten,* and *Addenden* are all used for numbers added. For operation purposes the numbers are not separated into groups of three figures, but when they are to be read the "periods" are used. In general, the horizontal arrangement of numbers in addition and the like is more common than with us. The magic square is skilfully used, and interest is increased by the introduction of some of the medieval methods of multiplying. The "ragged columns" of decimals, which we are coming to recognize are of no practical value to anyone, are seen, although no one has yet found a real case for an example like 24.5 + 20.897693. As with all who make much use of decimals (that is, where the metric system is current), the form 0.65 is used throughout instead of .65, and the expression $.65, sometimes found in our textbooks, but never in business documents, would not be tolerated. The multiplier is generally placed after the number multiplied—a usage long since abandoned here. Abbreviated ("Austrian") division is generally used, whereas with us it has never been at all common. Compound denominate numbers are used in angle measure much more extensively than here, as in cases like $13° 26' 30'' \times 7$. The work in fractions is generally limited to cases that are fairly reasonable, as in our modern books, but it would seem to be carried much farther than necessary either in a country where the metric system is the standard, or even in our schools. The work in G. C. D. and L. C. M., which has been reduced to a minimum with us, is about what would have been found in our books a generation ago. Examples like $5/8 \times 3/4 \div 10/13$ are not unusual in the book. One interesting feature is the connection of musical notes with the work in fractions, a scheme that may succeed in a city

like Vienna, but which has not been favorably received by us.

Passing over the rest of this book, the one for the "IV. Klasse" introduces algebraic notation and considers cases like $a + (b + c) = (a + b) + c = a + b + c$ and introduces "nests" of parentheses such as were found in our algebras twenty-five years ago but which have since been abandoned. The operations with algebraic expressions are related to those with numbers and are explained graphically as well as analytically. The work in polynomials is more extended than here. Prime numbers have a degree of attention no longer found with us, even in rather extended algebraic work. A case like the separating of 1792336896 into prime factors, while not difficult, would be looked upon by us as not worth the effort. The work in algebraic fractions is about what we had in the pre-war period.

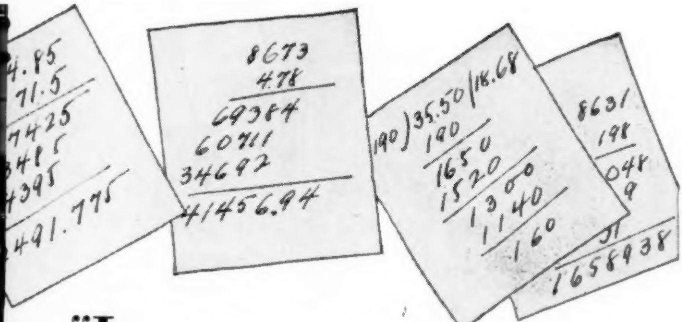
It would be of no service to make further comparisons. Enough has been stated to illustrate two essential differences between the work in the best schools of Austria and that in the best schools here. These are as follows:

(1) The Austrian schools are much more thorough than ours in the abstract work with number and with its algebraic representations.

(2) The American schools pay far more attention to the natural approach to the subject, to making the problems relate to the children's lives, and to the elimination of material that has no practical bearing upon anything with which the child is familiar. The reasons for these differences are probably (1) traditional, and (2) educational. We are still seeking to put every child through the high school, whether he is mentally equipped for it or not. Austria has a very fine instrument for sifting out the pupils who should follow less intellectual lines, and this is part of the sieve.

The third book represents what is being done in intuitive geometry, and is much more in harmony with what is attempted in our best junior high schools. It cultivates the motor activities as represented in geometric drawing, it deduces the rules for measuring simple objects, and it intuitively develops a number of the theorems of elementary geometry. This work is well worth examination by our teachers, giving as it does a number of methods of approach that are not commonly found in our textbooks. To show its catholicity, a map of part of New York City faces one of Karlsruhe—artistically somewhat to our disadvantage.

David Eugene Smith



TO ALL MEMBERS OF THE COUNCIL

We now have practically 5,000 members and we have set ourselves the task of securing 10,000 members by 1930. This means that we must double our present membership. However, this can easily be done if each teacher who is now a member will secure one other. From letters which reach us daily we know that there are still large numbers of teachers who know little or nothing of the work of the Council. We have accordingly published 20,000 circulars advertising the MATHEMATICS TEACHER and the yearbooks, copies of which we shall be glad to send to teachers who will agree to use them in situations where they may do good.

Finally, the Council will shortly issue as a supplement to one of the regular numbers of the MATHEMATICS TEACHER a register of members. It is important that each member of the Council send in at once his name, address, and teaching position as he wishes it to appear in the register. The address blank below, when properly filled out, should be mailed to the MATHEMATICS TEACHER, 525 West 120th St., New York City. Otherwise the name and address of each member will appear as it occurs on the mailing list.

PLEASE PRINT. MAKE EVERY WORD READABLE

1. **Name:** Mr. / Miss / Mrs. ....
   Last — First — Middle

2. **Degrees Received:** ....
   (When and Where)

3. **Official Position:** ....
   (Executive, Supervisor, Teacher—Sr. High, Jr. High, Elementary)

4. **Business Address:** ....

5. **Home Address:** ....
   No. — Street — City — State

6. **What Year Did You Become a Member?** ....
   (1920–To Date)

7. **Suggestions:** ....

....

Before Mailing Be Sure You Have Answered All Questions